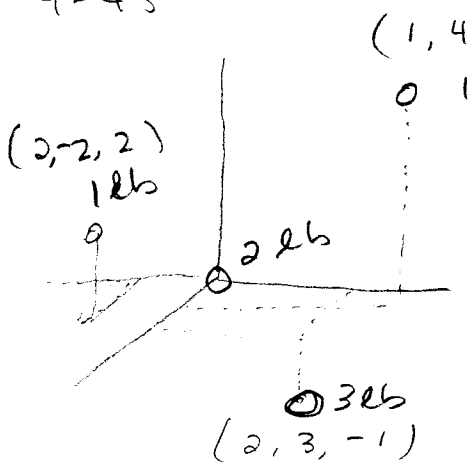
9-45



$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} = \frac{1(1) + 2(1) + 2(3) + 0(2)}{7}$$

$$\bar{x} = 1.29 \text{ ft}$$

$$\bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} = \frac{4(1) + (-2)(1) + 3(3) + 0(2)}{7}$$

$$\bar{y} = 1.57 \text{ ft}$$

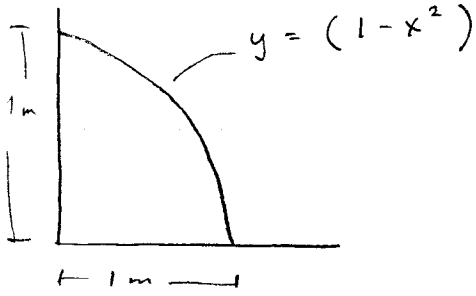
$$\bar{z} = \frac{\Sigma \tilde{z} W}{\Sigma W} = \frac{4(1) + (2)(1) - 1(3) + 0(2)}{7}$$

$$\bar{z} = 0.429 \text{ ft}$$

$$(1.29, 1.57, 0.49) \text{ ft}$$

HW handouts Section 9.1-9.2

9-1 Locate centroid **Solutions**



$dA = y\,dx = (1-x^2)\,dx$

$\tilde{x} = x$

$\tilde{y} = \frac{1}{2}y = \frac{1}{2}(1-x^2)$

} using dx as shown in figure (vertical)

using $y = 1-x^2 \Rightarrow x = \sqrt{1-y}$

$dA = \sqrt{1-y}\,dy \Rightarrow x\,dy$

$\tilde{x} = \frac{1}{2}\left(\sqrt{1-y}\right) \Rightarrow \frac{1}{2}x$

$\tilde{y} = y$

} using dy as shown in figure (horizontal)

using vertical dx data to solve:

$$\bar{x} = \frac{\int_0^1 \tilde{x}\,dA}{\int_0^1 dA} = \frac{\int_0^1 x(1-x^2)\,dx}{\int_0^1 1-x^2\,dx} = \frac{\int x - x^3\,dx}{\int 1-x^2\,dx} = \frac{\frac{x^2}{2} - \frac{x^4}{4}\Big|_0^1}{x - \frac{x^3}{3}\Big|_0^1}$$

$$= \frac{\frac{1^2}{2} - \frac{1^4}{4}}{1 - \frac{1^3}{3}} = \boxed{0.375\ \text{m}}$$

9-1 (cont)

$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA} = \frac{\int_0^1 \frac{1}{2}(1-x^2)(1-x^2)\,dx}{\int_0^1 (1-x^2)\,dx}$$

$$= \frac{\int_0^1 \frac{1}{2}(1-2x^2+x^4)\,dx}{\int_0^1 (1-x^2)\,dx} = \frac{\int_0^1 0.5 - x^2 + 0.5x^4\,dx}{\int_0^1 (1-x^2)\,dx}$$

$$= \frac{0.5x - \frac{x^3}{3} + \frac{0.5x^5}{5}\Big|_0^1}{x - \frac{x^3}{3}\Big|_0^1} = \frac{0.5(1) - \frac{1^3}{3} + \frac{0.5\,1^5}{5}}{1 - \frac{1^3}{3}}$$

$$= 0.4 \text{ m}$$

9-2

$dA = y\,dx = x^2\,dx$
$\tilde{x} = x$
$\tilde{y} = \frac{1}{2}x^2$

} using dx (vertical)

$dA = (1-x)\,dy = (1-\sqrt{y})\,dy$ ← total distance x-direction

$y = x^2 \Rightarrow x = \sqrt{y}$

$\tilde{x} = \frac{1+x}{2} = \frac{1+\sqrt{y}}{2}$

$\tilde{y} = y$

9-2 (cont)

using dx (vertical):

$$\bar{x} = \frac{\int_0^1 \tilde{x}\, dA}{\int_0^1 dA} = \frac{\int_0^1 x\,(x^2)\, dx}{\int_0^1 x^2\, dx} = \frac{\int_0^1 x^3\, dx}{\int_0^1 x^2\, dx} = \frac{\frac{x^4}{4}\Big|_0^1}{\frac{x^3}{3}\Big|_0^1}$$

$$= 0.75 \text{ m}$$

$$\bar{y} = \frac{\int_0^1 \tilde{y}\, dA}{\int_0^1 dA} = \frac{\int_0^1 \frac{1}{2} x^2\,(x^2)\, dx}{\int_0^1 x^2} = \frac{\int_0^1 0.5\, x^4\, dx}{\int_0^1 x^2}$$

$$= \frac{0.5\,\frac{x^5}{5}\Big|_0^1}{\frac{x^3}{3}\Big|_0^1} = 0.3 \text{ m}$$