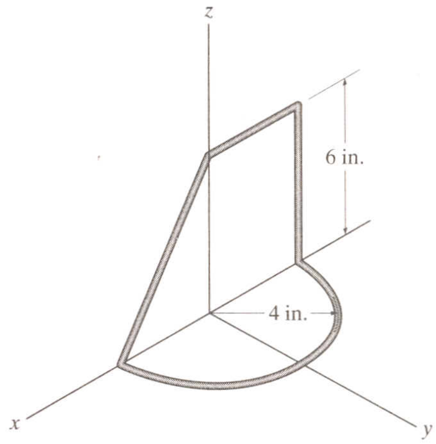
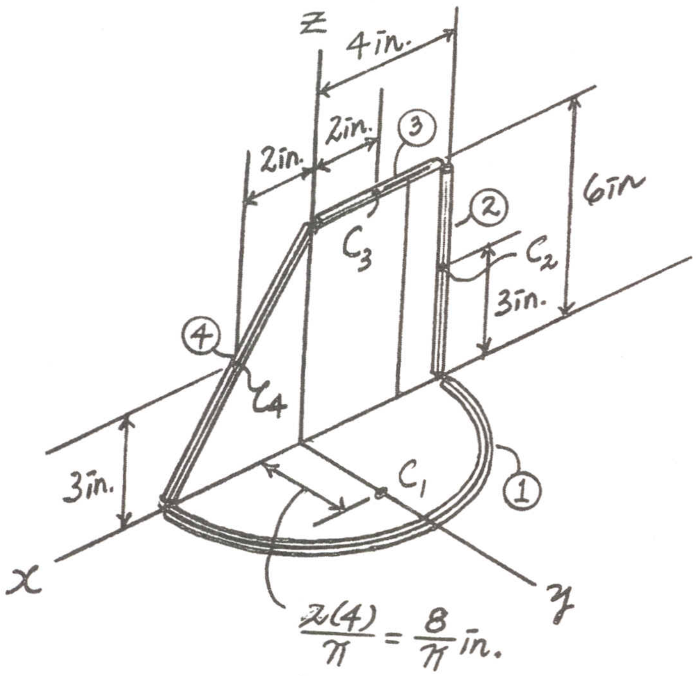
46. Locate the centroid $(\bar{x}, \bar{y}, \bar{z})$ of the wire.



$$\frac{\Sigma \tilde{x}L}{\Sigma L} = \frac{(0)\pi(4)+(-4)(6)+(-2)(4)+(2)\left(\sqrt{4^2+6^2}\right)}{\pi(4)+6+4+\sqrt{4^2+6^2}} = \frac{-17.58}{29.78} = -0.590 \text{ in.}$$ **Ans.**

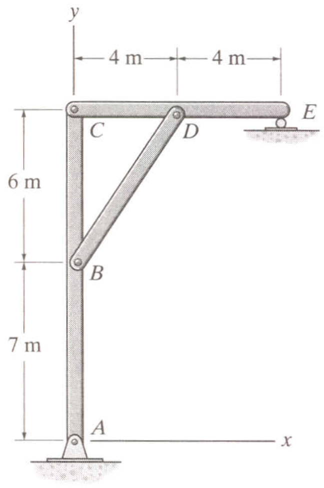
$$\frac{\Sigma \tilde{y}L}{\Sigma L} = \frac{(8/\pi)\pi(4)+0(6)+0(4)+0\left(\sqrt{4^2+6^2}\right)}{\pi(4)+6+4+\sqrt{4^2+6^2}} = \frac{32}{29.78} = 1.07 \text{ in.}$$ **Ans.**

$$\frac{\Sigma \tilde{z}L}{\Sigma L} = \frac{(0)\pi(4)+3(6)+6(4)+3\left(\sqrt{4^2+6^2}\right)}{\pi(4)+6+4+\sqrt{4^2+6^2}} = \frac{63.63}{29.78} = 2.14 \text{ in.}$$ **Ans.**



(a)

9–50. Each of the three members of the frame has a mass per unit length of 6 kg/m. Locate the position $(\bar{x}, \bar{y})$ of the center of mass. Neglect the size of the pins at the joints and the thickness of the members. Also, calculate the reactions at the pin A and roller E.



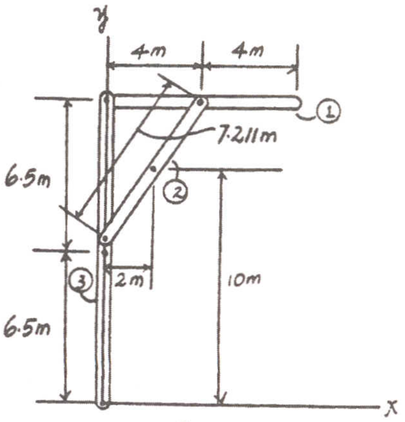
Centroid **:** The length of each segment and its respective centroid are tabulated below.

Segment	L(m)	$\tilde{x}$(m)	$\tilde{y}$(m)	$\tilde{x}L$(m²)	$\tilde{y}L$(m²)
1	8	4	13	32.0	104.0
2	7.211	2	10	14.42	72.11
3	13	0	6.5	0	84.5
Σ	28.211			46.42	260.61

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}L}{\Sigma L} = \frac{46.42}{28.211} = 1.646 \text{ m} = 1.65 \text{ m} \quad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma\tilde{y}L}{\Sigma L} = \frac{260.61}{28.211} = 9.238 \text{ m} = 9.24 \text{ m} \quad \textbf{Ans}$$



Equations of Equilibrium **:** The total weight of the frame is $W = 28.211(6)(9.81) = 1660.51$ N.

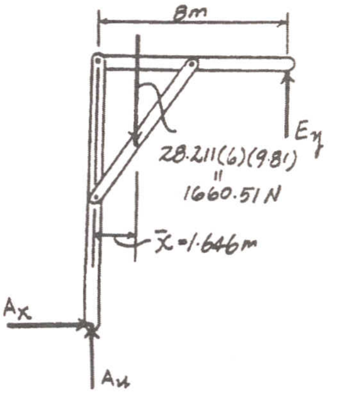
$+\Sigma M_A = 0;\quad E_y(8) - 1660.51(1.646) = 0$

$E_y = 341.55 \text{ N} = 342 \text{ N}$ **Ans**

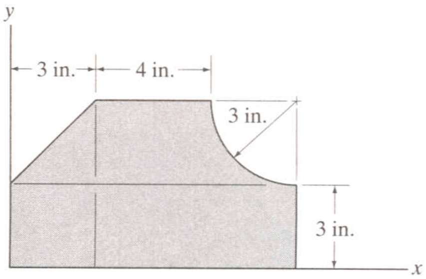
$+\uparrow \Sigma F_y = 0;\quad A_y + 341.55 - 1660.51 = 0$

$A_y = 1318.95 \text{ N} = 1.32 \text{ kN}$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x = 0$ **Ans**



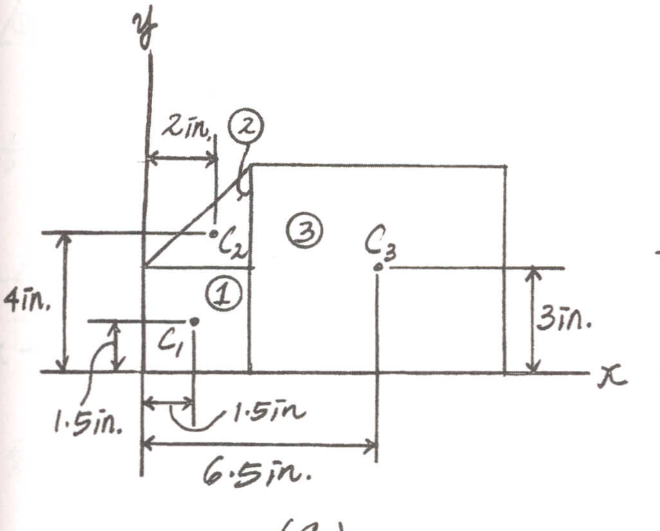
9–59. Locate the centroid $(\bar{x}, \bar{y})$ of the composite area.



Centroid: The centroid of each composite segment is shown in Figs. *a* and *b*. since segment (4) is a hole, its area should be considered negative.

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{1.5(3(3)) + 2\left(\frac{1}{2}(3)(3)\right) + 6.5(7(6)) + \left(10 - \frac{4(3)}{3\pi}\right)\left(-\frac{\pi(3^2)}{4}\right)}{3(3) + \frac{1}{2}(3)(3) + 7(6) + \left(-\frac{\pi(3^2)}{4}\right)} = \frac{233.81}{48.43} = 4.83 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{1.5(3(3)) + 4\left(\frac{1}{2}(3)(3)\right) + 3(7(6)) + \left(6 - \frac{4(3)}{3\pi}\right)\left(-\frac{\pi(3^2)}{4}\right)}{3(3) + \frac{1}{2}(3)(3) + 7(6) + \left(-\frac{\pi(3^2)}{4}\right)} = \frac{124.09}{48.43} = 2.56 \text{ in.} \qquad \textbf{Ans.}$$



(a)

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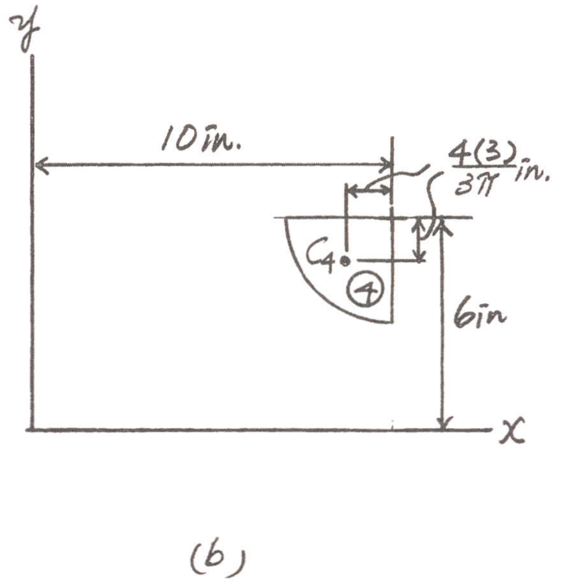
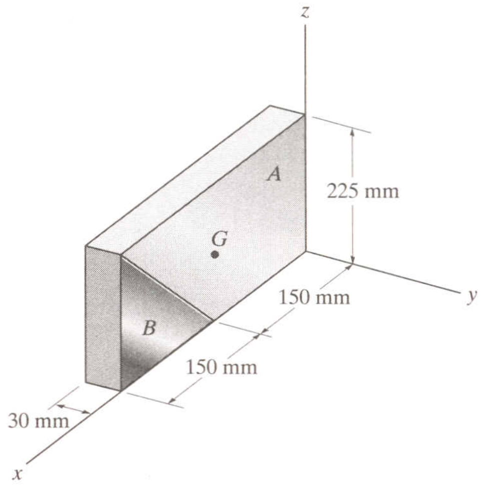


(b)

•9–65. The composite plate is made from both steel (A) and brass (B) segments. Determine the mass and location $(\bar{x}, \bar{y}, \bar{z})$ of its mass center G. Take $\rho_{st} = 7.85\ \text{Mg/m}^3$ and $\rho_{br} = 8.74\ \text{Mg/m}^3$.



$$\Sigma m = \Sigma \rho V = \left[8.74\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right] + \left[7.85\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right]$$

$$+ [7.85(0.15)(0.225)(0.03)]$$

$$= [4.4246(10^{-3})] + [3.9741(10^{-3})] + [7.9481(10^{-3})]$$

$$= 16.347(10^{-3}) = 16.4\ \text{kg} \quad \textbf{Ans}$$

$$\Sigma \tilde{x} m = \left(0.150 + \frac{2}{3}(0.150)\right)(4.4246)(10^{-3}) + \left(0.150 + \frac{1}{3}(0.150)\right)(3.9741)(10^{-3})$$

$$+ \frac{1}{2}(0.150)(7.9481)(10^{-3}) = 2.4971(10^{-3})\ \text{kg}\cdot\text{m}$$

$$\Sigma \tilde{z} m = \left(\frac{1}{3}(0.225)\right)(4.4246)(10^{-3}) + \left(\frac{2}{3}(0.225)\right)(3.9471)(10^{-3}) + \left(\frac{0.225}{2}\right)(7.9481)(10^{-3})$$

$$= 1.8221(10^{-3})\ \text{kg}\cdot\text{m}$$

$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m} = \frac{2.4971(10^{-3})}{16.347(10^{-3})} = 0.153\ \text{m} = 153\ \text{mm} \quad \textbf{Ans}$$

Due to symmetry:

$$\bar{y} = -15\ \text{mm} \quad \textbf{Ans}$$

$$\bar{z} = \frac{\Sigma \tilde{z} m}{\Sigma m} = \frac{1.8221(10^{-3})}{16.347(10^{-3})} = 0.1115\ \text{m} = 111\ \text{mm} \quad \textbf{Ans}$$