Practice Exam Chap 8
Math 144

1. Find the exact value of $\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

addition formulas $\sin(s+t) = \sin s \cos t + \cos s \sin t$

$\sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

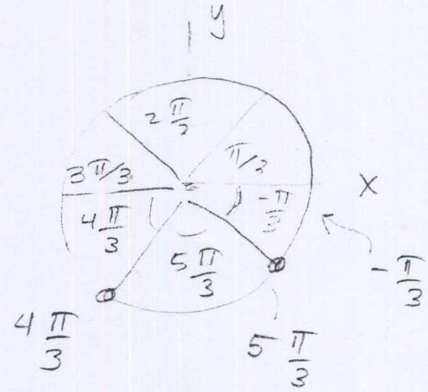
$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{(\sqrt{2})(\sqrt{3})}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

2. Find all solutions to $\sin x = -\frac{\sqrt{3}}{2}$ within the interval $[0, 2\pi]$

$x = \sin^{-1} -\frac{\sqrt{3}}{2} = -\frac{\pi}{3}$ (from calculator)

$\sin x = -\frac{\sqrt{3}}{2}$ so solution(s) in either (or both) Quad III or IV because that is where a sin value would be negative

$\boxed{\frac{4\pi}{3}, \frac{5\pi}{3}}$



3. Find all solutions of the equation $2\cos^2 x - 7\cos x + 3 = 0$ within the interval $[0, 2\pi]$. Express your answers in radians.

factor w/ calculator

$(\cos x - 3)(2\cos x - 1) = 0$

$\cos x = 3$ — not possible, Check w/ calculator

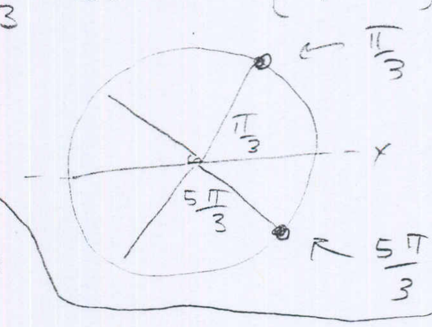
$2\cos x = 1$

$x = \cos^{-1}\frac{1}{2}$

$x = \frac{\pi}{3}$

$\boxed{\frac{\pi}{3}, \frac{5\pi}{3}}$

$\frac{\pi}{3}$ occurs with a positive cosine at $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ between $[0, 2\pi]$



4. If $\csc\theta = -3$ and $\pi < \theta < 3\pi/2$, compute sine $(\theta/2)$.

use half angle formula, $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos s}{2}}$

find $\sin\theta \Rightarrow$

$\csc\theta = \frac{1}{\sin\theta} = -3;\quad \sin\theta = -\frac{1}{3}$

$\cos\theta \Rightarrow \cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1-\left(-\frac{1}{3}\right)^2} = \sqrt{1-\frac{1}{9}}$

$= \sqrt{\frac{9}{9}-\frac{1}{9}} = \sqrt{\frac{8}{9}}\quad -\frac{2\sqrt{2}}{3}$

Because θ is in Quad 3, this is negative

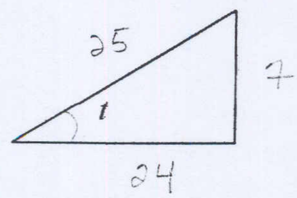
$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\left(-\frac{2\sqrt{2}}{3}\right)}{2}} = \pm\sqrt{\frac{\frac{3}{3}-\left(-\frac{2\sqrt{2}}{3}\right)}{2}}$

$= \sqrt{\frac{3+2\sqrt{2}}{2(3)}} = \sqrt{\frac{3+2\sqrt{2}}{6}\left(\frac{6}{6}\right)}$

$\boxed{= \frac{+\sqrt{18+12\sqrt{2}}}{6}}$

Because $\sin\frac{\theta}{2}$, $\pi < \theta < \frac{3\pi}{2}$ becomes $\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$ and the answer is positive because it is in Quad 2

5. Find cos ($2t$) given that t is an acute angle of a right triangle with side opposite of length 7 and side adjacent of length 24.



$\sqrt{24^2+7^2} = 25$

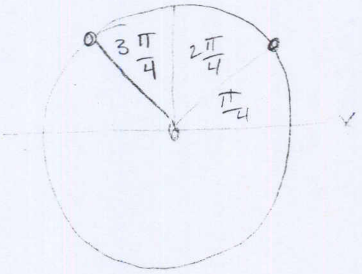
Double angle formula

$\cos(2t) = \cos^2 t - \sin^2 t$

$= \left(\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2 = \boxed{\frac{527}{625}}$

6. Determine all solutions to sin (θ) = √2 / 2.

$\theta = \sin^{-1} \frac{\sqrt{2}}{2} = \frac{\pi}{4}$



$$\boxed{\begin{array}{l} \frac{\pi}{4} + 2k\pi \\ \frac{3\pi}{4} + 2k\pi \end{array}}$$

7. Use a calculator to find all solutions to cos (t) = -0.567 in the interval [0, 2π]. Give your answers accurate to 4 decimal places.

negative cos means solutions are in Quad II and III

$t = \cos^{-1}(-0.567) = \boxed{2.1737}$

$= 2\pi - \cos^{-1}(-0.567) = \boxed{4.1095}$

third Quad because $-$ cos

8. Find all solutions to $5 \sin^2 \theta + 13 \sin \theta - 6 = 0$ in the interval $[0, 2\pi]$. Give answers accurate to four decimal places.

$$5 \sin^2 \theta + 13 \sin \theta - 6 = 0$$

$$(\sin \theta + 3)(5 \sin \theta - 2) = 0$$

$\sin \theta = -3$

can't happen w/ unit circle

$$\sin \theta = \frac{2}{5}$$

$$\theta = \sin^{-1} \frac{2}{5} = \boxed{0.4115}$$

$$\pi - \sin^{-1} \frac{2}{5} = \boxed{2.7301}$$

9. Evaluate the following: (do not have to show work).

a) $\sin(\sin^{-1} 2)$

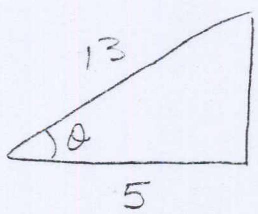
does not exist

b) $\text{sine}^{-1}(\sin(3\pi/2))$

$$\boxed{= -\frac{\pi}{2}}$$

10. Evaluate $\tan(\cos^{-1}(5/13))$. Show your work; your calculator answers the question, I need to see where the answer came from.

$$\cos^{-1}\left(\frac{5}{13}\right) = \frac{5}{13}$$ (from inverse identity p. 625)



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{opposite} = 5^2 + 13^2 = \text{opposite}^2$$

$$\sqrt{5^2 + 13^2} = 12$$

$$\boxed{\tan \theta = \frac{12}{5}}$$