Number Sense with Fluency Lesson Study

October 15, 2020

Research Question:

*"How can we develop students' number sense so that they, over time, become procedurally fluent, **automatize** of the basic facts and extend this understanding to other situations?"*

The lesson study team hypothesized that the following actions would be important elements in connecting number sense to fluency. Each hypothesis is listed below followed by the team's reflection.

Hypothesis 1 – From contexts to models as bridges to understanding

1. *Models which originate from context situations and which function as bridges to higher levels of understanding have a key role.*

- Students were modeling the situation with the blocks, and the students were using the blocks then to use strategies like doubling, doubling and halving within 3-d arrays.
- Students were referencing the oranges in boxes to make sense of combinations that would make sense.
- Students were using the model to think about "layers", which is a critical way of thinking about volume.
- Some students used the model to record their expressions and equations, and others had difficulty representing the model with the expression. This means that understanding that a number can be formed by several factors.

- Students connecting models to equations will later allow for vertical mathematization in terms of factors and the associative property.
- Doing explorations with the same number and finding different factors seems to have capacity to lead to fluency.
- Students were connecting between the model and the expressions to justify their actions.
- Many students used 3 factors after the congress, showing that the modeling of students' thinking on the expressions supported higher levels of understanding.

Hypothesis 2 – Guided opportunities to re-invent mathematics by doing it

Education should give students the "guided" opportunity to "re-invent" mathematics by doing it. (conferrals and modeling students' thinking)

- Many conferrals allowed students to move forward from what they had already produced, where students began constructing more combinations.
- The teacher modeled students' thinking on expressions from their 3-d arrays, allowing more students to readily represent their models on expressions or equations.
- The congress was an opportunity to share ideas that many students had with each other, that many other students tried as they continued the investigation after the congress.
- Questions the promoted students to generate new ideas, leaving ownership on the students of the ideas was useful in keeping students moving forward.
- Introduction of symbols and language in the moment it's needed (parentheses) is productive in supporting students' use of those symbols.

Hypothesis 3 – Horizontal and vertical mathematization

Horizontal and vertical mathematization are of equal value and this can occur at different levels of understanding.

- Some students had generalized the idea that they were investigating factors and were vertically mathematizing.
- Many students were referring to the oranges and boxes during the investigation and horizontally mathematizing.
- Many students were using symbols, but in varying degrees of the need to connect to the 3-d array.
- The models of the array and 3-d array allowed access to understanding mathematics more generally.

Hypothesis 4 – Talk moves to facilitate meaningful discourse

By listening to what others find out and discussing these findings, the students can get ideas for improving their strategies. Moreover, the interaction can evoke reflection, which is necessary to reach a higher level of understanding. (Strategic use of talk moves in the whole group discussion and in conferrals)

- In one pair, students were arguing about different ways to come up with more combinations by flipping their 3-D array, prompting reflection on whether they are different than other arrangements.
- During the congress, students shared the strategies they used but were selected strategically to make sure doubling and halving, the idea that numbers can be created from multiple factors, and the connection between the 2-d and 3-d array.
- After students discussed their ideas in the congress, they seemed to have more ideas on finding various combinations that they were willing to try and represent with expressions.
- The teacher used several turn and talks during the congress that allowed the dialogue ball to keep moving between the kids.

Hypothesis 5 – Building fluency from a foundation of understanding

Effective teaching of mathematics builds fluency (flexible and adaptive) with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

- Thinking about the mathematical development across the grades is important to consider in students' long-term trajectories.
- As students engage in the SMP's over time, they will have a productive disposition towards reasoning and problem-solving being confident to enter into challenging situations with flexibility.