Compensation Lesson Study

September 13, 2018

Research Question:

How can teachers pose purposeful questions to support students in developing and defending convincing arguments while making sense of problems?

The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to support students in developing and defending convincing arguments while making sense of problems. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 - Assess, celebrate, get underneath and challenge

> *Recognizing and celebrating what they've done or a big idea in their work describing where students are on the landscape, getting underneath to illuminate the big idea, and leaving them to develop a justification to present (walking away) is an effective questioning pattern that will advance students' reasoning and support them in developing and defending convincing arguments while making sense of problems.*

Hypothesis 2 - Using and understanding the landscape (learning trajectory) to drive questions

Using a learning trajectory to identify where students are in their understanding of big ideas and strategies, to celebrate what they've done and to guide what the teacher can push students towards will increase the teachers' capacity to pose purposeful questions that advance students' reasoning.

Shared Understandings/Reflection from the Team

Understanding and using the landscape is critical. By celebrating what a student has done the teacher has the opportunity to think about where the students are in the trajectory, forces the teacher to identify the students' next step/s and help formulate appropriate advancing questions. Getting underneath a students' strategy or idea helps dig out the math so that other students can see it and advance their thinking. The students that seem to be making the most progress in this class are the ones who seem to be listening to other students. Asking open-ended questions has been helpful for these students to learn to present and justify. Anticipating how students are going to think about the tasks and developing purposeful questions ahead of time is useful for the teacher. Providing regular opportunities for students to justify their thinking should support students in developing and defending convincing arguments.

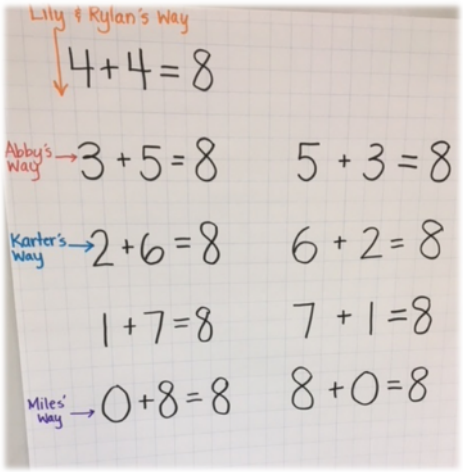
In the coming months, along with the items above, this group will attempt to provide regular opportunities for students to engage in tasks that promote reasoning and problem-solving, comparing different strategies or ways of thinking, using gallery walks before the whole group discussions, and developing an environment where students are actively listening to each other (re-voicing, having students re-voice each other and build on each other's thinking). Additionally, the team believes that developing a deep understanding of the landscape (learning trajectory) is a key ingredient in posing purposeful questions.

Compensation Lesson Study

September 18, 2018

Research Question:

How can teachers pose purposeful questions to advance students' reasoning?



The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to advance students' reasoning. Each hypothesis is listed below, followed by the team's reflection on each piece.

1. Assess, celebrate, get underneath and challenge

Recognizing and celebrating what they've done or a big idea in their work describing where students are on the landscape, getting underneath to illuminate the big idea, and leaving them to develop a justification to present (walking away) is an effective questioning pattern that will advance students' reasoning and support them in developing and defending convincing arguments while making sense of problems.

- It seemed difficult to get underneath students' ideas without correcting their mistakes and that this work will take time for teachers to develop in themselves.
- It took work on the part of the teacher to keep students grounded in the context, yet that work seemed to be critical for sense-making.
- It also seemed difficult to ask questions beyond assessing students' reasoning. Getting students' thinking out was more natural, but knowing what to get underneath and how to challenge them from where they are is difficult.

2. Using and understanding the landscape (learning trajectory) to drive questions

Using a learning trajectory to identify where students are in their understanding of big ideas and strategies, to celebrate what they've done and to guide what the teacher can push students towards will increase the teachers' capacity to pose purposeful questions that advance students' reasoning.

- The team believes it is very important to use the landscape to keep track of where and how students are moving on the landscape.
- The entire team grew in their knowledge and understanding of the landscape in this cycle by consistently listening to how students were thinking and trying to determine where that thinking is on the landscape.

3. Whole group discussion questions to move the community forward

Questions in the math congress should help to illuminate the big ideas underneath their strategies and move the whole class forward (up or across) the landscape.

- Using student work is a powerful way to drive the whole group discussion.
- When the teacher celebrates or identifies strategies that students are using during the whole group discussion, he/she reinforces the development of strategies and big ideas for all of the students.
- It seems that the math congress is a productive way to allow opportunities to engage in SMP3 (constructing viable arguments and critiquing the reasoning of others).
- It was clear that incorporating several "turn-and-talks" allowed opportunities for students to have rich discussions about the big ideas of equivalence and commutativity.
- Questions in the math congress should help to illuminate the big ideas underneath their strategies and move the whole class forward (up or across) the landscape. The teacher was consistently "narrating" what students were doing on the rack
- Keeping the students' ideas that are coming out grounded in the context is important for students' sense-making.

Doubling and Halving Lesson Study

September 19, 2018

Research Question:

How can teachers pose purposeful questions to advance students' reasoning?

The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to advance students' reasoning. Each hypothesis is listed below, followed by the team's reflection on each piece.

1. Assess, celebrate, get underneath and challenge

Assessing students' reasoning, celebrating what they've done or are trying to do, getting underneath the strategy, and leaving the students with a challenge could be an effective questioning pattern in small group conversations to advance students' reasoning.

- The team was pleased that students seemed to focus on how to they were thinking and their strategies, rather than the answer itself.
- Modifying the recording sheet to not restrict how students recorded their thinking gave more freedom in how students thought about the problems.
- The students used alternative strategies without being "taught" them ahead of time. Every student had a way to start on the problem, and every student had opportunities to learn something new.
- The task previous to this task prompted students to use doubling which may have contributed to their use of doubling in this task. Therefore, the sequences of the tasks that we use should promote development of the important big ideas and strategies.

- Students used the commutative property of multiplication without prompting, showing they were making connections.
- Students seemed to be coming up with alternative ways of thinking about the same problem, but need more to move them further in development.
- Asking students to develop convincing arguments about the structure of the mathematics is one way of advancing students and something this team would like to work on further.

2. Using and understanding the landscape (learning trajectory) to drive questions

Using a learning trajectory to identify where students are in their understanding of big ideas and strategies, to celebrate what they've done and to guide what the teacher can push students towards will increase the teachers' capacity to pose purposeful questions that advance students' reasoning.

- The team believes it is very important to use the landscape to keep track of where and how students are moving on the landscape.
- The entire team grew in their knowledge and understanding of the landscape in this cycle by consistently listening to how students were thinking and trying to determine where that thinking is on the landscape.

3. Whole group discussion questions to move the community forward

Posing questions in the whole-group discussion should be an attempt to generate new mathematics based on what students have already produced.

- In this case students did offer ideas about the commutative property of multiplication into the whole group discussion as well as worked towards a conjecture about halving and doubling.
- The conjecture that "we can half one of the factors and double the other factor while the product stays the same" is new idea that was generated from the students' thinking.
- The teacher asked students to write equations that would match the informal ideas that students had already produced (3 eights is the same as 3 x 8), which allowed more students to see the strategy of doubling and halving.
- The purpose of the whole group discussion is for the students to discuss with each other, not to share with the teacher.
- Since the focus was on the thinking and not "going over the answers" students didn't seem to feel like they were either right or wrong which allowed for risk-taking amongst students who frequently struggle.
- We will really need to work on making sure the whole group discussion is more than a show and tell of strategies, and reflecting on whether the mathematics moved forward in each day's whole group discussion.

Partial Products Lesson Study

September 21, 2018

Research Question:

How can teachers facilitate meaningful discourse in a way that will support students to develop and defend convincing arguments?

The lesson study team hypothesized that the following actions would be important elements in facilitating meaningful discourse in a way that supports students to develop and defend convincing arguments. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Teacher discourse moves

Using the teacher discourse moves of waiting, asking students to revoice what another student said, and creating opportunities to engage with another students' reasoning will support students to develop and defend convincing arguments.

> The teacher would specifically say, "I see that this student did…., but do you think they can do that again." , "can you restate what they did in your own words", "why do you think they did that", and continue to push what students said back to the rest of the students to think and talk about. When students would offer an idea, the teacher would ask students to listen and try out another students' thinking. The teacher allowed students to take ownership of their posters and defend or answer students' questions. The teacher discourse moves seemed to keep the whole group conversation student-centered.

Hypothesis 2 - Using and understanding the landscape (learning trajectory) to drive questions

Understanding and using a reliable learning trajectory (landscape) will support teachers' capacity to facilitate meaningful discourse by serving as a guide for what to focus the conversation on.

Using the landscape sheds light on what students are thinking and the disparities between students within their conversations. For example, two students were arguing about their strategies but it became apparent that one student had constructed unitizing and the other had not. By using the landscape it can help us understand how to push to the next level on the landscape. Deciding which models to use in the whole congress was determined by what seemed to be on the horizon for most students.

Hypothesis 3 – Assessing, celebrating, getting underneath and challenging

Posing questions in a structure of assessing, celebrating, getting underneath their strategies and challenging/advancing will be important in preparing students for the whole group discussion where they will be defending their arguments.

There were multiple instances where the teacher used this cycle fully. These instances seemed to provide a potential support for other students who were witnessing these conferrals. The teacher seemed to be able to get underneath the strategies that the kids were using and advancing students to higher level ways of thinking.

This cycle may have contributed to raising students' confidence in being willing to communicating their thinking and strategies. The kids seemed to be ready to defend themselves in the whole group because they have already done so in the conferrals with the teacher. It seems important to celebrate and turn their words into a more sophisticated mathematical statement through "are you saying…" and converts them into slightly more formal or generalized language, which allows students to use those ideas in other situations.

Hypothesis 4 – Gallery walk

Regularly incorporating a gallery walk before the whole group conversation will help to shift students' perspective to consider their audience when writing an argument.

It seems important for students to be silent during the gallery walk because it slows down the thinking for them to provide deeper comments and questions. Students know that other students are going to review their work. Students' questions from the gallery walk can be answered during the whole group discussion. Those questions they write during the gallery walk could be used as some of the questions in the whole group discussion.

Hypothesis 5 – Whole-group discussion

The goal of the whole-group discussion (math congress) is to help students see or learn something new that they did not see or learn before the discussion.

Students were definitely seeing other students as their audience, and some were making connections between representations without prompting. The teacher discourse moves seemed to help keep the discussion student-centered, where students were asking questions of each other and engaging with each others' reasoning. Some students were even making small steps towards understanding others' reasoning. In this lesson the discussion progressed toward an informal generalization based on what the students produced during the investigation.

Splitting the investigation on a separate day from the congress will help the teacher plan the congress. Using the questions that the kids write during the gallery walk in the whole group discussion is way to make their questions more meaningful. The questions that students ask in the gallery (rather than comments) should give insight to the teacher as to what the students are truly wondering about and should be tied to the whole group discussion. A clear goal for the congress based landscape is important for making the discussion meaningful.

Place Value Patterns Lesson Study

September 25, 2018

Research Question:

"How can teachers pose purposeful questions to advance students' reasoning?"

The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to advance students' reasoning. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Listening, Celebrating, Getting Underneath then Advancing

Using a structure comprised of listening first and/or noting what the students are doing or trying to do, celebrating some specifics of their ideas, getting underneath their strategies and suggesting further things to try a way that teachers can pose purposeful questions to advance students' reasoning.

- There were instances of celebrating and it seemed that the teacher was starting the conversations where students were at. The teacher would state what she thought the students were doing and ask their partners to explain.
- It seemed that the teacher needed more time in the conferrals for them to be more productive. However, there were many students who were moving forward when the teacher left the group.
- It seemed that when the teacher got underneath the students' thinking and either re-stated and/or recorded expressions or equations that matched what students were thinking students were advancing their reasoning. Both of these actions (re-voicing

and recording) seem to be a way of getting underneath and advancing students' reasoning.
- Listening to students first is very important to make sure we are building off of students' thinking to advance their reasoning.
- The teacher kept probing students to be precise with language and to continually stay grounded in the context.
- The teacher stayed focused on the big ideas and strategies without focusing on the mistakes that students were making or trying to fix the piece of mathematics they were producing.

Hypothesis 2 - Using and understanding the landscape (learning trajectory) to drive questions

A deep understanding and use of a reliable learning trajectory is important for teachers to pose purposeful questions to advance students' reasoning.

- The landscape seems to be important for planning and reflecting.
- It should also be used for making groups and partners of optimal mismatches.
- The landscape should drive a lot of teaching decisions that can be made before, during and after lessons.

Hypothesis 3 – Students using algorithms without understanding

When students are using algorithms prior to having constructed big ideas and strategies that should be earlier in development, teachers can use the same cycle above and push those students to develop an argument for why the algorithm works.

- No students tried to use standards algorithms in today's lesson so the team is unable comment.
- The team's plan for this scenario was to celebrate that they found an efficient way to solve the problem, press students to try to justify where each of the parts of the algorithm are in the problem, and develop an argument for why the procedure works.
- If students showed that they did not have a deep understanding of why the procedure works, the would have been asked to put that procedure to the side for a moment since we need to be able to convince others that it will always work, to work on a different strategy that they could convince others about, and over time, to keep thinking about why the algorithm works.

Hypothesis 4 – Gallery walk

Regularly incorporating a gallery walk before the whole group conversation will provide more opportunities for a meaningful whole-group discussion.

The team feels that the gallery walk does the following:

- Allows students to reflect on their own and others' thinking.

- Students see ideas that other students had in solving the problem.
- Gives students a chance to ask questions of each other.
- Allows an opportunity for students to see math as a process of continual revision.

Hypothesis 5 – Whole-group discussion

A whole group discussion can help students using inefficient strategies to consider other ways of thinking, making connections between different strategies, and to see something new that they didn't see before the discussion.

- This seems to be where the teacher starts to bring the different ideas together.
- The teacher at this point can highlight vocabulary based on what the students have already produced.
- This can also be a time where students compare different students' strategies.
- Turn and talks at strategic moments are helpful in the whole-group discussion.
- The conversation should be between the kids and dialogue passing between the kids more than from teacher to student.

Doubles and Skip-Counting Lesson Study

September 28, 2018

Research Question:

"How can teachers facilitate meaningful discourse to support students in engaging in SMP 1 and SMP 6?"

The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to engage in SMP 1 and SMP 6. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Using learning trajectories

Understanding and using a reliable learning trajectory will support teachers' capacity to facilitate meaningful discourse by serving as a guide for what to focus the conversation on.

- The trajectory could help to focus less on the task and more on the big ideas or generalizations that we want students to develop by working on the task.
- The team knew better where students were at and how to move them forward from where they are. We were able to develop questions that we wanted to ask in pairs and whole group to assure each student can see or generate new mathematics.
- The trajectory seems to help to manage all of the different strategies that students were using by narrowing down the number of different things that students are doing during the investigation.
- The trajectory can support teachers in choosing partners for an optimal mismatch of students who can push each other productively.

Hypothesis 2 – Assessing, celebrating, getting underneath then challenging

Posing questions in a structure of assessing, celebrating, getting underneath their strategies and challenging/advancing will be important in preparing students for meaningful discourse and engaging students in SMP 1 and SMP 6.

- Questions that draw attention to the structure of what's happening structurally underneath a students' way of thinking can drive meaningful discourse, even when the students are not the ones who explicitly bring that full idea to the front.
- Sometimes a student can create a model or use a strategy and not be aware of what they are doing (on the landscape), so questions should be used to "get underneath" a student's thinking to make what they're doing visible to themselves and other students is important to advance their reasoning.
- Getting underneath a student's thinking promotes SMP 6.
- It seems critical to assess students' reasoning first before we can ask questions to advance students' thinking.
- The teacher re-voiced student thinking numerous times in this lesson and used those as ways to get underneath student thinking, which seemed to unified students' partner work and their opportunities to attend to precision.
- Celebrating students' thinking in this cycle could contribute to a greater level of growth mindset and perseverance.

Hypothesis 3 – Using gallery walks before whole-group discussions

Gallery walks can help students orient to the thinking of others, give and get feedback about their strategies and arguments, supports students in writing mathematical arguments, analyze their own thinking and solutions in preparing for the whole-group discussion.

- The gallery walk seems to have given students' opportunities to develop their own questions that they can ask each other in the whole-group discussion.
- Students need practice and explicit help in knowing how to provide feedback and what is productive.
- The sentence starters seem to be helpful to student in developing a statement, but students seem to need more work with elaborating on that first comment.
- Some students were only looking to see if they agreed or disagreed with the answer.
- Limiting how many comments students should make in giving feedback to other students may help to allow for deeper comments.

Hypothesis 4 – Whole group discussion to generate new mathematics

The whole group discussion should be an opportunity for all students to see some new mathematics that they did not see before the discussion.

- Many students were able to see the idea underneath the students' ways of thinking during the congress, but were puzzled by the equation that showed the distributive property.
- The students were asking questions of each other when they were thinking about ideas that they hadn't thought about before, or when they were confused.
- The teacher sitting on the floor seem to allow the conversation to flow amongst the kids more than from the teacher to the students and vice-versa.
- Multiple students were adding on to each other's ideas through the teachers' questioning to make sense of another students' strategy or argument, re-voicing what students were saying, asking students' to build on each other's thinking which allowed students to speak with increasing levels of precision.

Doubles and Skip-Counting Lesson Study

October 3, 2018

Research Question:

"How can teachers pose purposeful questions to advance students' reasoning?"

The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to advance students' reasoning. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Listening, Celebrating, Getting Underneath then Challenging

Questioning that advances students' reasoning involves the teacher considering what the children are trying to do, celebrating and getting underneath what they've done or are trying to do, then challenging or upping the ante (ways to make the strategy more efficient, writing an argument to convince others of an idea, examining structure, patterns or relationships, generalizing beyond the problem).

- Pushing for efficiency is an opportunity to advance students' reasoning based on what they're already trying to do while students are investigating and can contribute to students beginning to generalize what's happening underneath their strategies.
- The teacher was pushing students to think about which numbers did not work.
- Leaving students with a justification to develop their own thinking further is a way to advance students' reasoning.

- The teacher was celebrating what students were trying to do and challenging them to think more about their ideas.
- Walking away may give students the space to think more deeply about their own ideas.

Hypothesis 2 - Using and understanding the landscape (learning trajectory) to drive questions

Teachers' deep understanding and use of a reliable learning trajectory is important for posing purposeful questions to advance students' reasoning.

- The team feels that it is important to understand a trajectory for asking questions to advance reasoning in an appropriate direction.
- It seems that it will take a significant amount of time to learn the landscape.
- Trying to figure out how to use the landscape to develop questions is a challenge to be thinking about over time.
- There is potential to use the landscape to inform instruction and share with parents where students are at and what is coming soon for them.

Hypothesis 3 – Pairing students in optimal mismatches

Pairing students in optimal mismatches allows students to productively support each other with new understandings.

- It seemed that students in this lesson were paired in optimal mismatches.
- The conversations within pairs seemed productive and most students were able to connect with each others' ways of thinking.
- The pairs where students were far apart on the landscape were not as collaboratively productive.

Hypothesis 4 – Gallery walk

Regularly incorporating a gallery walk before the whole group conversation will provide more opportunities for a meaningful whole-group discussion.

- Students were enthused about the gallery walk.
- Sentence strips might help develop deeper comments.
- It seems helpful for students to get to see other students' thinking in preparing students for the whole group conversation.
- When possible, we can give students time to revise their arguments prior to the congress.

Hypothesis 5 – Whole-group discussion

Questions in the whole group discussion should be more than asking students what they did and should get students to see something new that they did not see before the discussion partly through the teachers' choice and sequencing of models/strategies that lead up to the new idea.

- Students began to make generalizations about even numbers having partners towards the end of the congress. The teacher followed up with asking students whether that would always be true.
- The teacher recording of students' ideas was helpful in generalizing and advancing reasoning.
- The teacher kept asking questions about "will ___ always work?"

Slope Triangles Lesson Study

October 9, 2018

Research Question:

How can teachers facilitate whole-group discussions that support students in building on each others' ideas and support the development of number sense while addressing middle school level content?

The lesson study team hypothesized that the following actions would be important elements in supporting students to build on each other's ideas and support number sense while addressing middle school level content. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Purpose of the whole group discussion

Treating the goal of the whole-group discussion to help all students see new mathematics that they did not see before the conversation supports students in building on each other's ideas.

- This whole group discussion only had 7 minutes, which seems to be not enough time to have a significantly productive discussion from their arguments.
- Several students encountered new ideas in the whole group discussion that did not happen when they were working in small groups.
- Students' misconceptions were brought forward during the whole group discussion.
- Selection of which pieces of work is critical for a productive whole group discussion.
- Students needed more than 18 minutes to develop convincing arguments.
- Teachers need to be willing to provide enough time to develop arguments to contribute to the whole group discussion, potentially an entire class period.
- It also seems important for students to find similarities and differences between representations.
- A gallery walk where students comment on each others' representations/arguments could change the perspective of students that are brought into the whole group

discussion by considering that their work is to be seen as a contribution to the rest of the class or mathematical community.
- Students' written ideas before discussing with a group could contribute to more ideas on group presentations/arguments.
- It may be helpful to regularly quote students' conjectures to provoke investigations on generalized ideas to prove or disprove.

Hypothesis 2 – Use of the teacher discourse moves

Use of the teacher discourse moves (having students re-voice each other and creating opportunities to engage with each other's reasoning) will encourage active listening amongst students.

- The teacher was regularly asking students to add to another's reasoning, re-voicing and asking students to re-voice.
- The teacher validated/celebrated students' contributions to the whole group community which led to many students being willing to communicate in whole group discussions.
- Regular use of the teacher discourse moves seems to contribute to a productive disposition towards mathematics and classroom culture conducive to reasoning and problem-solving.
- Asking students to build on each other's reasoning during the whole group discussion by re-voicing what another student said, but asking students to explain why their reasoning is true is important for developing number sense and middle school level content because the ideas that work in middle school are based on a strong foundation of prior year's content.

Hypothesis 3 – Optimal mis-matching students

Use of optimal mismatches when pairing students for the investigations will allow students to push each other in small groups to reason, justify and generate new mathematics prior to the whole group discussion.

- Students in small groups were productive in this lesson because they were pushing each other along within their zone of proximal development.
- Most students were willing to contribute their ideas with each other.

Partial Products Lesson Study

October 18, 2018

Research Question:

How can teachers prepare for and facilitate meaningful whole group discussions that move all students forward?

The lesson study team hypothesized that the following actions would be important elements in preparing for and facilitating meaningful whole group discussions that move all students forward. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Listening, celebrating, getting underneath and challenging

Conferring with students by intently listening to what they are doing or trying to do, celebrating, getting underneath their strategy and challenging will be important for preparing students for a meaningful whole group discussion.

- The teacher conferred with a pair who re-grouped groups intensively using this structure and it seemed to support the students in being prepared about this particular big idea in the congress. The students who engaged in this discussion were willing and ready to engage with the whole group on this idea.
- Pushing another pair of students to articulate their thinking in a way that the other students will understand was supportive of the whole group discussion.
- There was a celebration of strategies that kids were trying in every conferral.
- Getting underneath students' strategies seems to be important, but also a challenge that will take time for us as teachers to learn how to do effectively.

Hypothesis 2 – Optimal mismatches of pairs of students

Optimal mismatches of students (pairing students that are close on the landscape but not in the same place) allows students to grow in development on the landscape before and after the whole group discussion.

- Some pairings of students seemed too close on the landscape, particularly students who started the task low on the landscape.

Hypothesis 3 – Pushing for generalizations

Pushing some students to generalize and convince others when students transition to investigating to preparing their poster/argument will make whole group discussions rich for all students.

- One pair of students was pushed to try to generalize doubling and halving with other numbers, and required a conferral where the teacher was getting underneath their strategy of regrouping groups.

Hypothesis 4 – Meaningful feedback in the gallery walk

Providing students with sentence frames in the gallery walk will allow students to share meaningful feedback and help all students consider the perspective of their audience when writing mathematical arguments.

- Several students used the sentence frames to generate comments during the gallery walk.
- It seems important for students to understand the purpose of the gallery walk.
- It seems that the gallery walk can or should allow for a more meaningful whole group discussion because students get an opportunity to see other strategies that may embolden their thoughts in the whole group discussion.
- The gallery walk seems to have given students' opportunities to develop their own questions that they can ask each other in the whole-group discussion.

Hypothesis 5 – Whole-group discussion

The Math Congress is meant to go beyond what they did (more of what you can do mathematically) helping to see new mathematics.

- There were several students who saw that re-grouping groups is a productive way of doing repeated addition, which should lead eventually to the associative property.
- Some students were beginning to understand how a ratio table can organize information and illustrate relationships.
- Some students changed their thinking that the 25 cents should be left as a unit instead of broken up.
- The conversation was largely happening amongst the kids where students were genuinely seeking to understand each other's thinking.

Partial Products Lesson Study

October 24, 2018

Research Question:

How can teachers support students to write their mathematical insights, strategies and arguments/justifications that deepens their own understandings and provides opportunities to move the entire class forward?

The lesson study team hypothesized that the following actions would be important elements in supporting students to write their mathematical insights, strategies and arguments/justifications. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Supporting the investigation

By asking questions that get underneath students' strategies that they're either doing or trying to do and challenge students to generalize their understandings teachers will support students to write their mathematical insights, strategies and arguments.

- Pushing students to try to prove that their strategy works, students didn't uncover the strategy until they tried to prove it.
- The challenge for students today was more about trying to prove their strategy works which lead to many a-ha moments that wouldn't have happened if student's task was to solve the problem.
- Shifting the conferrals from solving the problem to trying to prove that it works really deepens the students' understanding of the mathematical content. This shift seemed to

move students beyond answer-getting towards deepening their understanding of the mathematics.

- This section of the lesson today seemed to be where most students were making progress on the landscape.
- The teacher continued to tell students that their new goal once they've solved the problem was to now prove that the strategy works with other numbers.
- The team really pushed students to try to develop arguments as to whether their strategy would work with other numbers, and would always work. Some students did provide proof that the distributive property works. It might be that some students needed to investigate why the distributive property/partial products works before they could prove that it is generalizable.
- Students were really grappling with thinking about how they could possibly prove that partial products would "ALWAYS" work.
- The teacher kept referring to using their posters as a way to communicate to their audience. The students seemed to take it seriously that they would be showing their arguments to an audience.

Hypothesis 2 – Gallery walk

A gallery walk with helpful comments geared toward revision will help students to revise and strive for clarity to their argument/justification.

- Students were definitely responding to the comments that students had written as feedback.
- Some students were doing exactly what the feedback was asking them to do.
- Students certainly wanted to receive feedback from other students on their posters.
- Students may need some more work in recognizing that comments that they initially think are not helpful could be seen as helpful with some more thought.
- The gallery walk allowed students to see other ways of thinking about the same problem.
- Stating clearly that students would be doing a gallery walk (that there would be an audience) we were able to support students in writing their insights and strategies, and boosts students' desire to produce a quality poster.
- Many of the students' comments were not as much about the math but more about how they could present their ideas differently (adding more words, explain this part, etc).

Hypothesis 3 – Re-examine, revise and simplify

By giving students many experiences to re-examine, revise and simplify their ideas in order to make concise arguments for readers students will have opportunities to deepen their own understandings and move the whole class forward.

- The teacher asked at least one group to look at their strategy to be very clear in showing how they could clarify where the 24 twenty-fives were there.

- Students had many opportunities to re-examine and revise their their ideas from the beginning of the investigation, production of an argument, responding to the comments from the gallery walk and during the congress.
- It seems that the conferral time was valuable to give kids opportunities to re-examine, revise and simplify ideas and deepen students' understanding.
- Asking students to generalize their strategies and develop a convincing argument even if they are not going to present their arguments in the congress was helpful in moving students along the landscape and deepening their own understanding.

<u>Items the team will be working on until the next cycle include:</u>

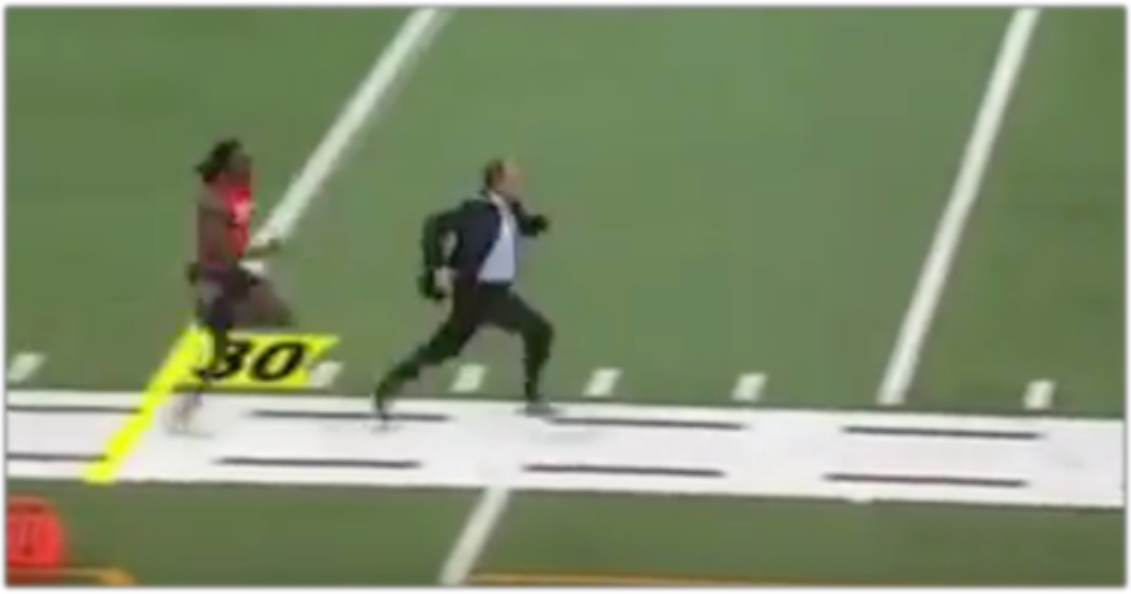
Pushing for generalization of the strategies
Making gallery walks the norm
During conferrals, not being satisfied that students have a strategy that they can share – pushing for more of "What do you want to convince your audience of?"
Making the expectation that the audience is students' classmates, and what they produce should always be considering their audience
Supporting students to look at the structure of the math while they are solving problems
mmediately start the revision of posters after the gallery walk and congress

Julio vs. Rich Lesson Study

November 7, 2018

Research Question:

How can teachers use and connect students' mathematical representations?



The lesson study team hypothesized that the following actions would be important elements in using and connecting students' mathematical representations. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Using students' representations rather than the teacher's

It is important for students to produce their own representation so that teachers can determine students' current levels of understanding.

- In one group, a student had written table but left it blank and said there has to be a time where they would meet because…
- Some students using calculators were trying to but struggled with connecting the variables within the calculator to the context.
- Students seemed to want to get right to equations that describe the situation.
- Several students were continually referencing the context from their representations to make sense and check for reasonableness, continually referring to units (seconds yards). (SMP 2)
- There was a high level of students engaging in SMP1 (making sense of problems and persevering in solving them).

- Some students were excited to see similarities in their ways of thinking displayed in the whole group discussion.
- Several students made sense of their own thinking through other students' representations.
- Students had ownership of the mathematics in part due to the fact that the discussion was centered on the students' (not the teachers') representations.
- Since the students' did not produce as much of what the team anticipated (tables), it seemed even more important to allow students to start by producing their own representations.

Hypothesis 2 – Purposeful questioning

Asking questions to assess and advance students' reasoning is critical for preparing students for the whole-group discussion where the teacher will use and connect students' representations.

- The students seemed to move into a different direction than the team anticipated making it difficult to ask assessing and advancing reasoning questions.
- Developmental trajectories might be useful in making our questions more productive.
- Pushing students back into the context is helpful in supporting students who are stuck.

Hypothesis 3 – "Convince the rest of the class"

Asking students to produce representations where the task is to convince the rest of the class.

- The teacher launched the task asking them to develop proof or evidence, but most students did not seem to be considering the perspective of the audience and would need some other work to do so.

Hypothesis 4 – Selecting and sequencing representations to generate new mathematics in the whole group discussion

Selecting/Sequencing should intentionally be used to allow discussion to arise around big mathematical ideas to allow the whole group discussion to generate new mathematics for all students.

- The teacher asked students to interpret each other's representations to move the thinking forward during the whole group discussion, making sense of the representation in terms of the context.
- Students were using other students' ideas to make sense of their own representations.
- Students were willing to use each other's representations to deepen their own understanding.
- The teacher was able to use representations that would allow for development of one of the identified big ideas and the discussion was moving in the direction of the identified big idea.

Hidden Triangles Lesson Study

November 8, 2018

Research Question:

How can teachers support productive struggle while maintaining cognitive demand?

The lesson study team hypothesized that the following actions would be important elements in supporting students in productive struggle while maintaining cognitive demand. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Clear goals for learning

Having a clear goal for students is maybe the most important idea, because unless you know what you want students to accomplish you don't know how to ask questions to move towards that goal.

- If you don't know what your goal is you don't know what questions to ask that will support their struggle, or you may ask questions that funnel students into your own way of thinking.
- Without a clear goal, you can take away the thinking from the students.
- Having a clear goal helps to know what ideas of the students to go after or not (Pythagorean Theorem in this lesson).

Hypothesis 2 – Questions based on students' thinking

Supporting productive struggle involves posing questions that are based on the students' thinking instead of trying to get the students' to think about the teachers' way to think.

- This task seemed to allow for students to struggle productively without much need for the teacher to be actively involved with advancing students' reasoning.
- Students were generating the mathematics and the struggle without much support from the teacher because they were required to justify their own claim.
- Having a task that has a low entry point with a lot of room to grow (justification) seemed to allow most of the students to advance their own reasoning.

Hypothesis 3 – Probing guidance to support productive struggle

Determining a student's thinking, encouraging self-reflection, and offering ideas based on the student's thinking is effective in supporting productive struggle while maintaining cognitive demand.

- When the teacher asks students to draw a picture to support their justification/claim, it seems to both determine a students' thinking and encouraging self-reflection.

Hypothesis 4 – Purpose of the whole-group discussion

All students should have the opportunity to collect more pieces of information about the issue of discussion and to understand the issue more deeply in the whole group discussion.

- Requiring students to provide a question for groups that presented made the writers of the question reflect on their own thinking.

<u>Further reflections from the lesson study team:</u>

- Some students were beginning to generalize towards the measurements of side lengths and angles immediately, while others were discussing tools needed to draw the triangle and precise vocabulary.
- Students had some difficulty developing justifications for their thinking and why their claim is true.
- The teacher was pushing small groups to justify their thinking along with a visual representation of their claims and justifications.
- Pressing students to justify their thinking may have supported students' willingness to listen to each other in the whole group discussion.
- Asking students to develop a question to pose to a group after they shared their claim and justification for the claim allowed students to develop their own ideas further.
- The group agreed that many of the students were engaged in productive struggle throughout the task.
- Students' thinking changed based on the questions that were given to groups from their presentations.

Skip-Counting and Doubles Lesson Study

November 13, 2018

Research Question:

How can teachers use purposeful questioning during students' investigations that supports productive struggle and prepares students for meaningful whole-group discussions?

The lesson study team hypothesized that the following actions would be important elements in teachers' use of purposeful questioning during students' investigations that supports productive struggle and prepares students for meaningful whole-group discussions. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Celebrating and "upping the ante"

Using the structure of considering what the children are trying to do (with a genuine interest in their thinking), celebrating the accomplishments you see, then upping the ante and challenging them based on what they are thinking and trying to do supports productive struggle and prepares students for a meaningful whole-group discussion.

- There were several students that were discussing even/odd before the investigation even began and the teacher allowed for discussion without trying to formalize.
- The teacher was consistently showing a genuine interest in students' thinking when conferring.
- The teacher was celebrating what the students did and upping the ante by asking for a counter-example.
- The students worked on the task for 35 minutes struggling productively.

- This structure of questioning seems to be connected to supporting productive struggle, but also the teacher walking away to give time to think and give space to think about the challenging question.
- The option for kids to choose a number to try seemed to be an entry point for all students.
- The teacher said that the students will work on a new investigation tomorrow that is connected to today's.
- It is important to anticipate what the students are going to do in order to think ahead of time what questions to ask and how to advance students' reasoning.
- The questions focus their ideas towards big ideas that students can share during the congress while still honoring what they are thinking.
- By questioning that supports productive struggle throughout the investigation, students are more invested in engaging the whole group discussion.

Hypothesis 2 – Developing the mathematician rather than fixing the sheet

Questioning needs to support development of the mathematician rather than finding the right answer, and support a growth mindset through targeted questions, not just "why" questions.

- Questions that start where students are at are critical without focusing on the mistakes, but letting the structure of math workshop be supportive of students fixing their mistakes.
- A focus on right answers, questions for kids to fix their mistakes promote answer-getting and against productive struggle.
- Teachers should focus on the strategies that students are working on or trying instead of fixing the sheet.

Hypothesis 3 – Understanding learning trajectories

A deep understanding of learning trajectories can be used as a guide for questioning and supports teachers in using the structure of questioning in hypothesis #1.

- Trajectories are a support for teachers to know what to push on when questioning.
- Students' strategies were evident in the string of related problems.
- The Rekenrek seemed to allow students to make their strategies visible.

Hypothesis 4 – Purpose of the whole-group discussion

A meaningful whole-group discussion should generate new mathematics and help the students see something new that they didn't see before the discussion.

- The students were building off of each other's thinking.
- Some students did grow in their understanding that skip-counting is adding 2 each time.
- The team identified a few big ideas to go after in the whole group discussion.

- The whole group discussion is a time where we can use the landscape to connect strategies and models to new big ideas.
- Optimal mismatching students was supportive of meaningful whole group discussions as they'd been pushing each other before the whole-group discussion.
- The discussion allows students to grow from each other's ideas.
- There were many conjectures and generalized statements that students were developing in the discussion.

Introducing Fractions Lesson Study

December 6, 2018

Research Question:

How can teachers prepare for and facilitate meaningful whole group discussions that move all students forward?

The lesson study team hypothesized that the following actions would be important elements in preparing for and facilitating meaningful whole group discussions that move all students forward. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Listening, celebrating, getting underneath and challenging

Conferring with students by intently listening to what they are doing or trying to do, celebrating, getting underneath their strategy and challenging will be important for preparing students for a meaningful whole group discussion.

- There were many instances of the teacher celebrating, getting underneath and challenging.
- When the teacher used this structure and then walked away, there were many instances where the students continued thinking and talking about the challenging question for extended periods of time.
- Students were engaged in each other's work, struggling together.
- The teacher does not need to get to every pair because the conferrals will take some time to be powerful.
- It's important to give students some time to get started.

- Students who were struggling were consistently brought back to the context and students persevered throughout the hour of working on the task.
- Students were constructing viable arguments and critiquing the reasoning of others throughout the task.
- This is an important structure to use to support the investigation.

Hypothesis 2 – Optimal mismatches of pairs of students

Optimal mismatches of students (pairing students that are close on the landscape but not in the same place) allows students to grow in development on the landscape before and after the whole group discussion.

- Today's pairings seemed to be optimal mismatches because the students were able to connect with each other's ideas.
- Some students were using the investigation to listen to each other's ideas more than to share their own ideas.
- The pairings may have also forced some students who typically wouldn't be active in sharing their ideas to put forth their thoughts during the investigation.

Hypothesis 3 – Pushing for generalizations

Pushing some students to generalize and convince others when students transition to investigating to preparing their poster/argument will make whole group discussions rich for all students.

- This seems important for teachers who have primarily a procedural understanding of the mathematical content.
- In this lesson most of the generalizing and convincing occurred in the congress.
- It is important to highlight the purpose of the posters for congress so students know what to focus on, which could be just one particular idea not necessarily everything they did.

Hypothesis 4 – Meaningful feedback in the gallery walk

Providing students with sentence frames in the gallery walk will allow students to share meaningful feedback and help all students consider the perspective of their audience when writing mathematical arguments.

- Students need to have ongoing experience with the gallery walks to be more specific and helpful with their comments.
- Students were wanting more specific comments on their own work from their peers, but it also prompted students to make revisions to their work.
- Students seemed unsure what to write and copied other students' comments.
- Students were more concerned with providing feedback than with reading a mathematical argument.
- Gallery walks are important for the whole group discussion and should be done regularly.

- The students should know that this is a time for them to write something genuine to enhance the whole-group discussion for the sake of the entire mathematical community.

Hypothesis 5 – Whole-group discussion

The Math Congress is meant to go beyond what they did (more of what you can do mathematically) helping to see new mathematics.

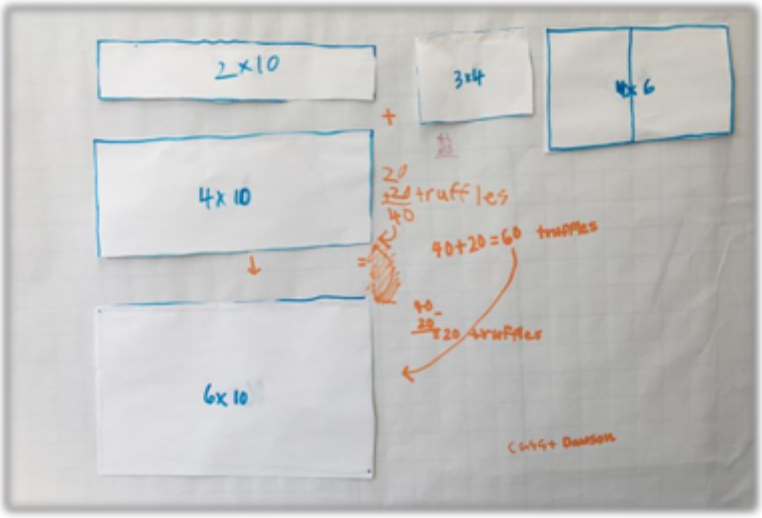
- The conversation was certainly focused on the big ideas that the team identified.
- The presentations moved beyond show and tell of what they did to more about the mathematics.
- It is important to allow time for the students to respond to each others' ideas when the big ideas are being brought out by students.
- Teachers should acknowledge that students are often still progressing even though they may appear to be disengaged in the discussion. Many instances kids appear to be checked out but repeatedly engage back in the discussion.
- Pushing for generalization of the strategies and big ideas should be a focus during the congress.

Doubles and Skip-Counting Lesson Study

January 14, 2019

Research Question:

How can teachers pose purposeful questions to advance students' reasoning?



The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to advance students' reasoning. Each hypothesis is listed below, followed by the team's reflection on each piece.

1. Assess, celebrate, get underneath and challenge

Recognizing and celebrating what they've done or a big idea in their work describing where students are on the landscape, getting underneath to illuminate the big idea, and leaving them to develop a justification to present (walking away) is an effective questioning pattern that will advance students' reasoning and support them in developing and defending convincing arguments while making sense of problems.

On multiple occasions, the teacher used this cycle of questioning in the small group discussions during the investigation, particularly getting underneath students' strategies and challenging. Several times the teacher asked students to convince the rest of the class of their ideas. It seemed clear that several groups of students were advancing their reasoning because of the teacher's questioning, looking for relationships between arrays, and connecting to multiplication.

If we are having longer, deeper conferrals it would be important to make sure over time we are conferring with all students. There seems to be a window in which to confer with each student if we have a sequence of tasks that scaffolds student learning in small increments.

2. Using and understanding the landscape (learning trajectory) to drive questions

Using a learning trajectory to identify where students are in their understanding of big ideas and strategies, to celebrate what they've done and to guide what the teacher can push students towards will increase the teachers' capacity to pose purposeful questions that advance students' reasoning.

Without a learning trajectory, advancing students' reasoning would be very difficult. It seems more reasonable to understand the landscape in terms of what the students actually produce in the task/investigation. The landscape seems useful in interpreting student thinking from the investigation to use or emphasize in the congress.

3. Whole group discussion questions to move the community forward

Questions in the math congress should help to illuminate the big ideas underneath their strategies and move the whole class forward (up or across) the landscape.

The team felt like we should pay attention to accountability of student engagement in the congress, possibly with math journals to record in during the discussion. In this congress, students were exposed to the need for identifying the rows and columns in numeric expressions. Having thought about the big ideas that we want to draw out in the congress is helpful in sequencing the discussion. There is no one right way to hold a congress, but the goal is for the whole community to move on the landscape.

Doubles and Skip-Counting Lesson Study

January 15, 2019

Research Question:

How can teachers pose purposeful questions to advance students' reasoning?

The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to advance students' reasoning. Each hypothesis is listed below, followed by the team's reflection on each piece.

1. Assess, celebrate, get underneath and challenge

Recognizing and celebrating what they've done or a big idea in their work describing where students are on the landscape, getting underneath to illuminate the big idea, and leaving them to develop a justification to present (walking away) is an effective questioning pattern that will advance students' reasoning and support them in developing and defending convincing arguments while making sense of problems.

Many students seemed intrigued by the task when the teacher was conferring using this cycle. The teacher walking away after using this structure seemed to support students in staying interested in the task and moving forward. The questions that the teacher asked in this case pushed students away from creating more "examples" or models and towards the structure of the mathematics (How are we going to know whether a number is going

to work without drawing the model?). Bringing students back to the context seemed powerful in supporting students to continue moving forward during the investigation. It is important in the challenge questions to ask students to investigate "why" something is working before they are asked to convince others.

2. Using and understanding the landscape (learning trajectory) to drive questions

Using a learning trajectory to identify where students are in their understanding of big ideas and strategies, to celebrate what they've done and to guide what the teacher can push students towards will increase the teachers' capacity to pose purposeful questions that advance students' reasoning.

A deep understanding of the landscape (trajectory) is essential in asking questions to advance students' reasoning, as well as to reflect on a lesson and student thinking after the lesson.

3. Whole group discussion questions to move the community forward

Questions in the math congress should help to illuminate the big ideas underneath their strategies and move the whole class forward (up or across) the landscape.

Using turn and talks was powerful in determining what individual students are thinking, giving students a voice, allowing each other to listen to other students' thinking and ask each other questions. The congress sheds light as to where students are on the landscape, what understandings are still fragile, and pushes students to think more generally. Many questions in the congress should not have just one right answer and invite kids to build off each others' thinking.

Fractions Lesson Study

January 29, 2019

Research Theme:

Supporting students to develop and defend convincing arguments while making sense of problems

The lesson study team hypothesized that the following actions would be important elements in facilitating meaningful discourse in a way that supports students to develop and defend convincing arguments while making sense of problems. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Modeling student thinking

If we model students' thinking and the processes that they use they will have objects to discuss and can examine their logic.

- Students found value in adding to their own representations and arguments during conferrals when the teacher was modeling student thinking on the double number line.
- One student knew ¼ as being 1 divided into 4 equal parts which helped expose her thinking, but seeing it modeled in a slightly more sophisticated model (double number line) helped to clarify what she was thinking and to examine her own logic.
- In pairs where students are spread a bit apart in terms of the landscape, it might be important to start where the student that may be having a more difficult time to model that students' thinking first.

- Emphasizing consistent models (double number line) from various tasks within and across units helps to bridge different lessons, students' ideas from previous days/work and relate it to what will happen in that day's lesson.
- Providing a model that linked to students' thinking clearly allowed students to advance in their reasoning.
- Modeling students' thinking in conferrals in ways that has been modeled in other situations (number strings, previous tasks) allows those students to see that their reasoning is advancing, while also allow students to use that model of their thinking as proof.

Hypothesis 2 – Conferring to support argumentation

Conferrals that include the teacher modeling student thinking and using questions that get underneath students' strategies, identifying those as ideas that their audience needs to know, enables students greater capacity to later revise their written argument to re-sequence their ideas.

- The teacher used the "rulers" to support students in seeing the numbers they were identifying for various lengths.
- There were numerous times that the teacher asked students to consider what their audience needs to know, which seemed to help students treat their posters as way to communicate to the rest of the other students in the class.

Hypothesis 3 – Assessing, celebrating, getting underneath and challenging

Suggesting for students to use "if, then" statements to develop an argument during conferrals and the congress

- Part of the task was for students to use "if, then" statements was part of the task, but that was not enough to get students to actually use them.
- It may be that the teacher could use "if, then" statements as regular practice with students so they are more inclined to understand the usefulness in developing an argument.

Hypothesis 4 – Re-examine, revise and simplify ideas

Children need many experiences reexamining, revising and simplifying their ideas in order to make concise arguments for readers.

- Students' comments during the gallery walk caused other students to reflect on their ideas, and try to revise and clarify them.
- As students had more time and were pressed to make generalizations from their strategies, their statements became more simplified (every 3 inches is ¼ of a foot, every 4 inches is 1/3 of a foot, and the bigger the bottom number the smaller the piece is).
- Opportunities for students to develop arguments and posters over time allows students opportunities to re-visit those ideas.

Open Number Line Lesson Study

January 30, 2019

Research Question:

Supporting students to develop and defend convincing arguments while making sense of problems

The lesson study team hypothesized that the following actions would be important elements in supporting students to develop and defend convincing arguments. Each hypothesis is listed below, followed by the team's reflection on each piece.

1. Re-examine, revise and simplify

Children need many experiences to re-examine, revise and simplify their ideas in order to make concise arguments for readers/audience. (after the gallery walk)

- One student did make revisions to their poster after reading the comments from the poster, but many did not.
- Allowing time for these experiences would be necessary for development over time.
- Part of simplifying ideas could mean providing students time to add clarification for why a generalized statement works (12 tens and 5 more is 125).

2. Gallery walks

Using gallery walks to allow students to interpret others' ideas in terms of their own work will allow more access for students to re-examine, revise and simplify their ideas.

- Students were given time to review the comments they got from other students during the gallery walk.
- Some students revised their thinking after reading other students' comments.
- The more visual representations may provide more access to K-1 students who may have difficulty reading each others' posters and comments.

3. Modeling student thinking

We need to model students' thinking and the processes they use so they have objects to discuss and can examine their logic.

- During the congress, it seemed difficult to connect the t-chart to the number line based on what students were describing as strategies to locate places on the number line.
- Many students counted by ones in the investigation when measuring, but counted by tens in the congress.
- Several students made more general statements during a follow-up conferral where the teacher modeled student thinking on the open number line.
- The model teachers use to model student thinking should be connected to other tasks within a unit and connected to promoting development.

Constant Difference Lesson Study

February 5, 2019

Research Question:

How can teachers use purposeful questioning during students' investigations that supports productive struggle and prepares students for meaningful whole-group discussions?

The lesson study team hypothesized that the following actions would be important elements in teachers' use of purposeful questioning during students' investigations that supports productive struggle and prepares students for meaningful whole-group discussions. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Celebrating and "upping the ante"

Using the structure of considering what the children are trying to do (with a genuine interest in their thinking), celebrating the accomplishments you see, then upping the ante and challenging them based on what they are thinking and trying to do supports productive struggle and prepares students for a meaningful whole-group discussion.

- The teacher was able to celebrate in many conferrals after listening to what students were doing or trying to do, no matter where they were on the landscape.
- The teacher modeled student thinking on the number line on several occasions.

- It was evident that when the teacher starts with where students are at, models their thinking, asks a question to get underneath or challenge their thinking, students are be advancing their reasoning.
- It seems important for the teacher to be explicit about the big ideas (getting underneath) that students are coming up with during the conferral and that they understand what they are now thinking more about after the conferral.

Hypothesis 2 – Shifting the conferral towards proof

Questioning in conferrals can shift to prove and convince the rest of the students that their strategy works.

- Even though students were asked to show why something works, or to convince the class of an idea, they still seemed to simply re-tell what they did or show their strategies.
- Asking students to prove or convince is not enough for getting students to prove to the rest of the class. Possibly, referencing the context to include in their proving might be helpful.
- Staying grounded in the context (encouraging students to use the context to help prove their ideas) might give greater access to proving and convincing.

Hypothesis 3 – Writing for an audience

Gallery walks should help students understand they are writing for an audience, to reflect on whether their ideas would make sense to somebody else, and clearly share their ideas. Feedback in the gallery walk from students to students should show that they can contribute to each others' thinking.

- When the teacher provides comments on students' posters during the gallery walk might help students to understand what types of comments might be more helpful.
- Students did want to respond to the teacher's comments on their posters.
- Teachers can consider using sentence frames for providing comments.

Hypothesis 4 – Purpose of the whole-group discussion

A meaningful whole-group discussion should generate new mathematics and help the students see something new that they didn't see before the discussion.

- We want opportunities for all students to see the models/strategies that students are discussing.
- Pushing the dialogue around between the kids is important for keeping students in the conversation, talking about each others' ideas.
- Asking students to share out in the whole group what their partner is thinking from the turn and talks might be helpful in making the congress meaningful.

Linear Measurement Lesson Study

February 19, 2019

Research Question:

"How can teachers facilitate meaningful discourse to support students in engaging in SMP 1 and SMP 6?"

The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to engage in SMP 1 and SMP 6. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Celebrating, getting underneath, challenging

Posing questions in a structure of assessing, celebrating, getting underneath their strategies and challenging/advancing will be important in preparing students for meaningful discourse and engaging students in SMP 1 and SMP 6.

- In conferrals, the teacher was always trying to start with what the student was doing or trying to do.
- When students were stuck in the abstraction, asking questions that refer the students back to the context.
- It seems important for the teacher to be persistent in articulating many times while getting underneath students' strategies.
- It was helpful for the teacher to point out "this is something your audience needs to know" during the conferrals for the students to attend to precision.

- Students seemed to understand that they had an audience (other students) to try to convince.

Hypothesis 2 – Modeling student thinking

We need to model students' thinking and the processes they use so students have objects to discuss and can examine their logic.

- Questions that draw attention to the structure of what's happening structurally underneath a students' way of thinking can drive meaningful discourse, even when the students are not the ones who explicitly bring that full idea to the front.
- Sometimes a student can create a model or use a strategy and not be aware of what they are doing (on the landscape), so questions should be used to "get underneath" a student's thinking to make what they're doing visible to themselves and other students is important to advance their reasoning.
- Getting underneath a student's thinking promotes SMP 6.
- It seems critical to assess students' reasoning first before we can ask questions to advance students' thinking.
- The teacher re-voiced student thinking numerous times in this lesson and used those as ways to get underneath student thinking, which seemed to unified students' partner work and their opportunities to attend to precision.
- Celebrating students' thinking in this cycle could contribute to a greater level of growth mindset and perseverance.

Hypothesis 3 – Positioning with teacher discourse moves

Intentionally using the teacher discourse moves with students that have "low status" will increase their chances for participating meaningfully in the whole group discussion.

- The teacher had written statements the students made in previous lessons to start this lesson and publicized them early in the lesson, which seemed to be of high interest to the students and have a lot potential to elevate their own status in their own minds.
- Students were re-voicing each other throughout this lesson without much prompting from the teacher.
- The teacher re-voiced and quoted students multiple times throughout this lesson, and it seemed to support students' willingness to take risks.

Hypothesis 4 – Whole group discussion to move the community forward

The whole group discussion should be an opportunity for all students to see some new mathematics that they did not see before the discussion.

- Students in this classroom were frequently asking questions of each other without prompting from the teacher while also focusing on the mathematics.

- Students were trying to make sense of each others' models during the congress, asking questions of each other and engaging with each others' reasoning.
- The teacher's use of Wait Time II, waiting after a student makes a comment, allowed students to have more discussion between each other than had the teacher interevened sooner.

Fractions Lesson Study

February 22, 2019

Research Question:

"Supporting students to develop and defend convincing arguments while making sense of problems"

The lesson study team hypothesized that the following actions would be important elements in supporting students to develop and defend convincing arguments while making sense of problems. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Specifying what to convince others of

Leaving students in a conferral with a specific big idea (written) to try to convince the class on the poster about may be important in teaching students how to write an argument.

- Writing the claim/s that we want students to try to prove that describe the generalization the lies underneath the strategies.
- This also seemed to push some students away from just trying to get the answers to the problems.
- Many students still seemed to have difficulty in trying to prove the claim even after it was written.
- Focusing on different types of argument can be a focus of the congress (where the mathematical ideas could become a tertiary focus).

- Writing the claim on a separate paper during the conferrals might be useful in shifting students from trying to answer the problem in the context to generalize more about the mathematics.
- It's important that the claim is connected to what the students are thinking about and within their zone of proximal development, yet challenging.

Hypothesis 2 – Modeling students' thinking

We need to model students' thinking and the processes they use so they have objects to discuss and can examine their logic.

- This practices seems important for supporting students to develop arguments, but we need to be sure that the modeling is connected to students' real thinking.
- The landscape should be helpful in determining what students are trying to do, which will help the teacher decide on what the modeling could look like to match the students' thinking.
- Modeling their thinking should be helpful in supporting kids to organize their thinking.

Hypothesis 3 – Reorganizing ideas

Providing proof to a community requires a reorganization of ideas into a chain of logically connected statements by analyzing and resequencing their ideas in search of convincing chains of reasoning (rather than re-telling). (Since this is true…then this would also have to be true…statements)

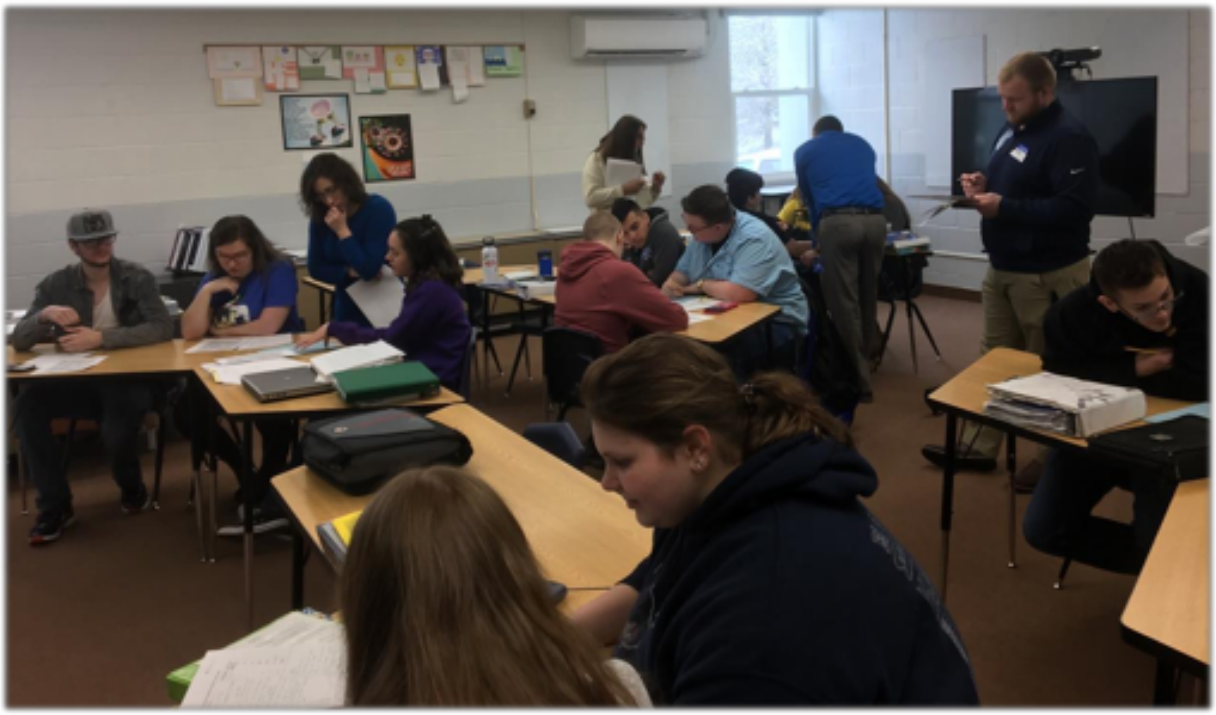
- Reflections in hypotheses 1 and 2.

Proof and Proving Lesson Study

February 26, 2019

Research Question:

How can teachers support students to value understanding other students' ways of thinking and why (the mathematical structure of) mathematical strategies and procedures work?



The lesson study team hypothesized that the following actions would be important elements in supporting students to value each other's understanding and proving. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Observe, conjecture, revise, prove

Using tasks that allow students to (1) make observations before they engage in proving activities; (2) make or revise conjectures, and (3) provide a mathematical argument or proof.

- In this case, the students were given conjectures that the lesson study team believed were connected to the students' strategies to solve the problems, but it wasn't clear that students saw the connection to that conjecture and their strategy.
- Students were creating new conjectures throughout the task, even if it wasn't the same conjecture that they were given.

- Students in the whole group discussion were responding to each others' ideas, challenging each other, questioning each other and clarifying each others' ideas without prompting from the teacher.
- Several generalized statements were made by students during the whole group discussion but the teacher kept asking students what they thought about those statements.
- The context allowed students to always having something to fall back on to make meaning of the quantities.

Hypothesis 2 – Modeling student thinking

Teachers can consider modeling student thinking to support the development of an argument and highlight the generality of students' ideas.

- Proving activities allow teachers to have opportunities to introduce formal notation attached to students' informal ideas.

Hypothesis 3 – Make sense of each other's proofs

It is vitally important that students have regular opportunities to make sense of one another's proofs – not only for developing skills in generating and evaluating proofs, but also for helping them understand the necessity of doing proof in classrooms.

- The students in this lesson did a gallery walk to evaluate each others' arguments.
- The students need reminders that the purpose of doing a gallery walk is to provided feedback to the authors to better their arguments/proofs.
- A few things students provided for feedback prompted good reflection on their own arguments.

Hypothesis 4 – Student rubric for proving

Guiding them to use the rubric might help them focus on the representation or type of argument, catalyzing a more productive discussion than if they simply dismiss an argument because the answer happens to be incorrect.

- The rubric prompted students to add examples or other features to their written arguments.
- The rubric helped students understand what they could revise in their argument to make it more like proof.
- It didn't seem like students used the rubric to write feedback during the gallery walk, but it also seems helpful for the arguments to be displayed in the whole group discussion while students could reference the rubric.
- Teachers should be cautious the rubric isn't used primarily to evaluate each others' proofs so as to prevent students from wanting to contribute in the future.
- Students were able to rely on the rubric to understand more about what is meant by proof and as a support to generate discussion between themselves.

Hypothesis 5 – Conjecture wall to support inquisitiveness

While possibly not to be planned for, "conjecture walls," on which students can post conjectures that they discover but may not be able to prove at the time, can be useful classroom resources for encouraging mathematical inquisitiveness.

- Students wrote several statements in their arguments that could have been used on a conjecture wall.
- Some statements were also made in the whole group discussion that could be used on a conjecture wall.

Constant Difference Lesson Study

February 27, 2019

Research Theme:

"Supporting students to develop and defend convincing arguments while making sense of problems"

The lesson study team hypothesized that the following actions would be important elements in supporting students to develop and defend convincing arguments while making sense of problems. Each hypothesis is listed below, followed by the team's reflection.

Hypotheses

1. Students need many opportunities to re-examine, revise and simplify their ideas in order to make concise arguments for readers.
2. We need to model students' thinking and the processes they use so they have objects to discuss and can examine their logic.
3. Encourage the use of "if-then" statements that start with rules established within the community.

Team's reflection

Mentoring students in conferrals could include

- What you're trying to convince your audience of
- What do you think the community already knows that you could start your argument with?

- What do you think the community of mathematicians needs to know to be convinced of that idea that's not yet on your poster?

Using sentence frames for students to use in their comments during the gallery walk helps to support students in developing convincing arguments.

The comments in the gallery walk help students to see that they are writing arguments for an audience because they can see that what they are writing either does or does not make sense to somebody else.

Teaching students in the moment of developing arguments pointing out what are features of good arguments might be useful in developing better arguments over time.

Students need support in deciding what big idea they are trying to convince everybody of and recording that in their argument for their audience. ("That's something your audience needs to know.")

Documenting statements for students to prove seems helpful but it's imperative that the statement is connected to what students are actually thinking rather than just a push to do something else that they cannot see the connection to what they were already thinking.

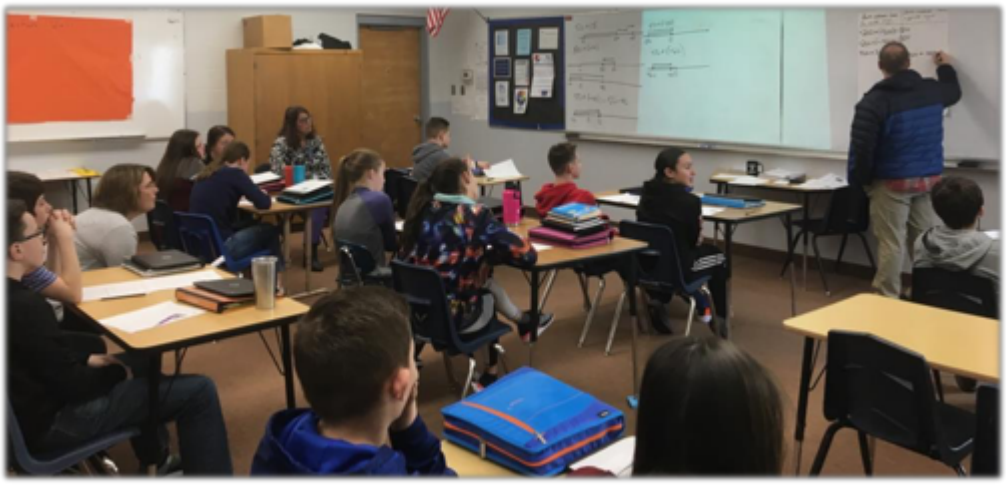
We are not trying to get all the kids to prove the same thing or get the same "it" at the same time.

Adding/Subtracting Signed Numbers Lesson Study

March 11, 2019

Research Question:

How can teachers support students to look for and make use of structure (generalize)? Does this have an impact on students' valuing each others' ways of thinking?



The lesson study team hypothesized that the following actions would be important elements in supporting students to engage in SMP 7 and SMP 8. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Strings of related problems

Starting a task where students can experience special cases or simpler forms in order to gain insight into its structure or solution.

- During the string of related problems, students frequently referenced problems and ideas that were brought up in the previous lesson, connecting across problems.
- The number string set the tone for the rest of the class.
- Students were exposing many conjectures and big ideas during the string of related problems.
- Students were very comfortable verbalizing their thinking in this setting.
- The teacher consistently used the Teacher Discourse Moves to set up lots of opportunities for students to engage with each other's reasoning.
- The modeling of student thinking during the string of related problems likely impacted students' capacity to conjecture, generalize and problem-solve.

Hypothesis 2 – Generalizing and attending to precision

Having students generalize will allow opportunities for students to use increasingly precise language and symbolism.

- After several ways of describing the mathematical structure, students automatically were using increasingly precise language.
- Students were trying to use language of opposite, magnitude, direction, difference, value to describe the actions and structure of positive and negative numbers.
- Students consistenly made generalizations about the relationship between addition and subtraction, including with negative numbers.

Hypothesis 3 – Building a conjecture wall

While possibly not to be planned for, "conjecture walls," on which students can post conjectures that they discover but may not be able to prove at the time, can be useful classroom resources for encouraging mathematical inquisitiveness.

- The conjectures on the wall from previous lessons were referenced during this lesson and slightly revised based on new discoveries in this lesson.
- The conjectures don't have to be correct, or complete, and seems to encourage mathematical inquisitiveness.
- Students were reminding the teacher to credit the student who made the comment/conjecture.
- The teacher should keep in mind that statements on the wall are going to get increasingly precise amongst the students.

Individual take-aways

Using conjecture walls to quote students and push revision and precisions. (Suzanne)
Using a conjecture wall will support students in proof and proving. (Robbin)
The team is interested in using number strings to support SMP 7 and 8.

Facilitating Whole-Group Discourse Lesson Study

March 12, 2019

Research Question:

How can teachers facilitate whole-group discussions that support students in building on each other's ideas and support the development of number sense while addressing middle school level content?

The lesson study team hypothesized that the following actions would be important elements in facilitating meaningful whole-group student discourse. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Building on student's ideas

Treating the goal of the whole-group discussion to help all students see new mathematics that they did not see before the conversation supports students in building on each other's ideas.

- There were a number of students that were connecting their own ideas and strategies to what was happening in the whole group discussion. Students were able to see their own thinking in others' thinking or models.
- Students were asking questions of each others' thinking even after a preliminary discussion between themselves. Students were asking for clarification of each others' ideas.
- Students' use of calculators to examine relatinships in the whole group discussion aided in their understanding of the connection between unit rate and the constant of proportionality.

Hypothesis 2 – Teacher discourse moves

Use of the teacher discourse moves (having students re-voice each other and creating opportunities to engage with each others' reasoning) will encourage active listening amongst students.

- The teacher in both small and whole group extensively used re-voicing and waiting in this lesson, and students were building on each others' thinking and asking questions of each other.
- Many students were trying to make sense of each others' thinking throughout the discussion.
- Students were probing each others' thinking in the small group discussions.
- Students were frequently asked to re-voice each other, which prompted students to come up with new ideas and generate new discussion.
- The students were listening actively with each other and there was significant use of the teacher discourse moves.
- The lesson study team believes that the Teacher Discourse Moves will support students to listen actively to each other in lessons beyond this lesson.
- The small group discussion served to allow the teacher to choose specific students to share ideas in the whole group discussion.

Hypothesis 3 – Re-voicing with sentence frames

Having students re-voice each other with sentence starters that keep the ideas of the author of the original statement.

- Even though students were given sentence frames, students didn't necessarily use them in their discussions.
- The team wonders if it might be more use-able by students if there were less frames to use (2 or 3).
- Students seemed to want to start their discussion with what they did themselves, as opposed to thinking about each others' thinking.
- The team feels we could, in the future, ask students to look only at one or two other students' ways of thinking, and not their own.
- There may be specific sentence frames that can be used for specific lessons or portions of lessons.

<u>Individual Take-Aways</u>

Having a list of "big ideas" for the teacher for every lesson is critical to keep the conversations focused on the most important mathematics. (Melissa)
We don't have to wait for every student to fully construct the ideas prior to starting the whole group discussion. (Melissa)
Whole group discussions are important for allowing the whole class to grow, and the Teacher Discourse Moves are helpful in supporting kids to build on each others' ideas. (Elaine)
Without whole group discussions would be short, minimal and less powerful. (Elaine)

The whole group discussion is also useful for developing common language (increasingly precise language – constant of proportionality, unit rate, etc). (Jennifer)
Wait time is critical for supporting kids to build on each other's thinking. (Melissa)
Considering the developmental levels of all students during the whole group discussion should connect to the use of the Teacher Discourse Moves. (Jennifer)

Posing Purposeful Questions Lesson Study

March 13, 2019

Research Question:

How can teachers use purposeful questions to advance students' reasoning from their current level of understanding?

The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to advance students' reasoning from their current level of understanding. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Re-voicing students' ideas

When the teacher re-voices a student, asking students whether that matches what they modeled or wrote captures the intent of the student and advances students' reasoning.

- When the teacher re-voices students, it gives others students in the group a chance to think more about what a fellow student said.
- When re-voicing students, the teacher can also connect the student's thinking back to the context and help the student see the fallacy in his/her own reasoning.
- The teacher can re-voice (get underneath) differences of ideas between students to provide a catalyst for a new discussion amongst students.
- There were many instances where the teacher re-voiced a student that allowed an opportunity for students to make shifts in their thinking.
- While re-voicing, the teacher gets an opportunity really listen to what students are thinking that allows him or her to understand what to get underneath and challenge.

Hypothesis 2 – "Focusing" patterns of questioning

A focusing pattern of questioning involves the teacher's honoring what the students are thinking by pressing students to communicate their thinking clearly and asking them to reflect on their thinking and the thinking of their classmates.

- Pressing students to communicate their thinking clearly helps students reflect on their own thinking.
- This pattern of questioning seems to be a way to support productive struggle.
- When the teacher's goal is to make the task easier for students we are not supporting productive struggle.
- Using this pattern of questioning over time should allow students to recognize that the teacher is not trying to lead them down a desired path (funneling). This should also help students to see that the teacher is trying to have a discussion with the student, rather than trying to get them to a certain answer.
- The launch of the task was simply used to get the task out and clarify what the question is. This supported them coming up with their own ideas and try to make sense of each other's ideas.
- Funneling, in part, comes from trying to get all students at the same place at the end of the lesson, no matter where they are starting at.
- Getting deeper into one problem is better than superficially speeding through answers.
- A focusing pattern of questioning supports SMP 6 (attending to precision with language).

Hypothesis 3 – Assessing then advancing questions

Asking questions in a sequence of assessing students' reasoning (more open) and advancing students' reasoning (more targeted) is a structure to use when posing purposeful questions.

- It seems important for the teacher to mention to students that he/she will be coming back to check on their progress in a few minutes, which can support accountability for rich discussion when the teacher walks away.
- Even with students who already have an answer, assessing reasoning can uncover gaps in understanding but pushing for a justification can support advancing reasoning.
- If you want to intentionally advance students' reasoning, the teacher must assess students' reasoning first and listen intently to their thinking.
- Walking away sends the message that the teacher believes the students are capable of carrying the task forward without direct support.