Subtraction Lesson Study

September 5, 2019

Research Question:

"Supporting students to develop and defend convincing arguments"

The lesson study team hypothesized that the following actions would be important elements in supporting students to develop and defend convincing arguments. Each hypothesis is listed below followed by the team's reflection.

Hypothesis 1 – Re-voicing and having students re-voice each other

The teacher can re-voice or ask students to re-voice each other to focus the conversation on an important idea or acknowledge a noticing.

- Re-voicing students from the string (including recording the student's statement on the board) regarding working with tens instead of ones was later used in the congress.
- Re-voicing students and then pushing students to turn and talk about that statement allowed students to have conversation on an important idea/noticing.
- During the string, one student said I could just use the other problem there and then subtract one from it. The teacher re-voicing made the warrant that supported the student's claim more

explicit by saying "So you know that the answer to the last problem was __ so you could just use this one to help you with the next one."
- One new student asked another student to clarify his thinking, potentially as a result of some of the re-voicing or passing the dialogue ball amongst the kids.

Hypothesis 2 – Inviting noticings of relationships and properties

The teacher can invite noticings of relationships, properties and regularities to move toward building arguments that are more general.

- During the string, students were prompted to notice relationships between the problems and were able to articulate relationships between them.
- The teacher asked what the students noticed about the 3 equations that represented the situation on the number line, which shifted the students' language from talking about how they solved the problem to more general relationships about the connection between addition and subtraction.
- The students were asked to describe relationships that they saw within subtraction, but the teacher extended the conversation to think also about addition which they were not thinking about prior to that prompting/question.
- Asking students what each number in the equations represented in the context (year dad was born, this year, age) gave an opportunity to develop language that could be used in further discussions about the relationships between the equations.
- When students are asked to make arguments it's helpful in clarifying their thinking.

Hypothesis 3 – Questions to shift attention towards general relationships

Questions should shift students' attention towards general relationships through celebrating, getting underneath students' thinking and challenging, and through opening the conversation for all to share their observations.

- The sequencing of students' thinking in the whole-group discussion was part of celebrating productive thinking that most students could connect to.

Hypothesis 4 – Relationships that emerge from students' own work

It is important to build on opportunities for attending to relationships as they emerge from students' own work and comments during the investigations and the congress.

- The student's work and thinking were the centerpiece of the whole group discussion. Students were talking about each other's thinking.
- Towards the end of the discussion, the teacher posed the question "Can subtraction can be thought of as addition?" focusing students on the relationship between addition/subtraction.

This question, however, came from the equations that students constructed while thinking about each others' strategies.
- By building on the students' thinking and explicitly referring to the students' thinking, it was the "right time" for students to begin to examine the relationships that lie underneath the strategies that they are using. Because it was built on what students were currently developing, it did not seem to the students that it was a different or separate idea to think about.

Hypothesis 5 – Clarify, justify, explain, critique and elaborate

The teacher's questions should allow students to clarify, justify, explain, critique and elaborate.

- Students were pressed to justify their strategies when their thinking was novel that many students had not yet thought about.
- Teachers should consider not pressing students to justify "everything", but rather new ideas that the community needs to think more about.
- Students can be provided opportunities to critique their own and each other's thinking and claims.

Hypothesis 6 – Modeling students' thinking

If we model students' thinking and the processes that they use they will have objects to discuss and can examine their logic.

- The teacher modeled students' thinking with open number lines and equations during the congress helped students to talk about more general relationships that exist between addition, subtraction and the meanings of the quantities in subtraction equations.
- The number lines seemed to help students recognize the meaning of subtraction as difference. They were referencing the number lines that were models of student thinking when thinking and talking about conjectures that were being made about subtraction being related to addition and subtraction being thought of as distance between 2 numbers.

<u>Individual teacher take-aways</u>

- I will try to brainstorm questions ahead of time that clarify, justify, explain, critique and elaborate. (Mimi)
- I will try to have students "elaborate" more of their initial thinking by asking them to "say a little bit more". (Connie)
- Having flexibility in the sequence of the students' thinking in the whole group discussion by identifying the most important big ideas to let students have discussion around and having the patience to let the students have discussions between themselves. (Joe)
- I want to limit the questions that I ask to focus more on the most important big ideas so that we can spend more time on the most important mathematics. (Val)
- I want to think about how to transfer this structure in ELA and think more about how argumentation is a process not a product. (Jill)

- I want to figure out how to use questions to shift students' attention towards general relationships. (Ryan)

Relating Tables and Graphs Lesson Study

September 9, 2019

Research Question:

"Supporting students to develop and defend convincing arguments"

The lesson study team hypothesized that the following actions would be important elements in supporting students to develop and defend convincing arguments. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Re-voicing and having students re-voice each other

If the teacher re-voices students, or asks students to re-voice each other's ideas, we can structure and focus the conversation on an important idea and potentially have students add on to each other's ideas.

- There were several instances of the teacher re-voicing students' ideas and asking them to re-voice each other both in small and whole group discussions.
- Re-voicing allowed the teacher to make the students' ideas explicit, allow other students to think about those statements and thoughtfully make their language more precise.
- It could be that the students were re-voicing each other (without the teacher asking them) because of the teacher's habit of re-voicing and asking students to re-voice each other on a regular basis.
- Asking students to re-voice each other in whole group discussions could help students to recognize that the ideas they bring to that discussion can also be built upon and revised, rather than seeing the whole group discussion as a "test" or final product that must be complete and perfect.

Hypothesis 2 – Questions to shift student's attention towards general relationships

Questions should shift students' attention from computation to general relationships, and prompt for clarification, justification, explanation, elaboration and critique to support unpacking and development of ideas (building blocks of argument)

- Asking students to *write* a definition for proportional relationships allowed them to contribute their informal ideas and continually revise them and shifted their attention away from computation towards general relationships.
- Asking students to *write a definition in groups* allowed the students to simply contribute ideas and revise them as they needed without having individual ownership over that newly crafted definition.
- The question of "what did you notice about the graph", even though the teacher did not say it because it was written on the paper in the task, the students shifted from computation towards general relationships by talking about "always" and "why".

Hypothesis 3 – Noticing and articulating mathematical ideas

The teacher should support students to notice and articulate mathematical ideas to develop the habit of noticing relationships, properties and regularities because noticing and articulating pave the way toward building an argument.

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Individual team-member take-aways

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Partial Products Lesson Study

September 13, 2019

Research Question:

"Supporting students to develop and defend convincing arguments"

The lesson study team hypothesized that the following actions would be important elements in supporting students to develop and defend convincing arguments. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Modeling students' thinking

We need to model students' thinking and the processes they use so they have objects to discuss and can examine their logic.

- Students were working on partial products but writing 2 equations separate from each other. The teacher wrote the equation as 2 equivalent expressions and the students seemed to be able to connect their thinking to that equation.
- In another instance, the teacher took a student's thinking and co-developed a ratio table.
- During the congress, equations were written to connect to the students' thinking which allowed students to describe what was happening as doubling and halving.

Hypothesis 2 – Celebrating, getting underneath and challenging

Conferring with students by intently listening to what they are doing or trying to do, celebrating, getting underneath their strategy and challenging will be important for preparing students for a meaningful whole group discussion.

- There were many instances of the teacher celebrating, getting underneath and challenging during the conferrals.
- Understanding the landscape is useful in looking for what students are trying to do instead of finding what they are not doing that we want to "fix".
- The teacher was pointing out what students were trying to do and push them from where they are on the landscape.

Hypothesis 3 – Re-voicing, asking students to re-voice each other

Re-voicing students' ideas and/or asking students to rephrase what one of their peers just said in their own words allows students to hear some of the important statements several times to understand their significance and can lead to asking students whether the other students agree or disagree.

- The teacher was re-voicing and asking students to re-voice each other during the congress and in the conferrals.
- When students had opportunities to re-voice or hear important ideas more than once it seemed to allow more students to participate in the discussion.
- Re-voicing while modeling student thinking and then asking students to re-voice each other seems to support development.
- Re-voicing allows opportunities for the teacher to introduce more formal mathematical language at a time when there is a need for it.
- This practice might support students in doing re-voicing and clarifying each other's thinking without the teacher prompting them to do so in the future. It seemed that some students were starting to do this in their conversations on their own.

Hypothesis 4 – Re-voicing, asking students to re-voice each other

The teacher's role is to help students notice and articulate mathematical ideas, support the development of mathematical language and help students unpack their ideas whether incomplete or even inclusive of faulty solutions.

- It seemed that the modeling of student thinking is useful in helping students to notice and articulate mathematical ideas.
- The students were focused on the mathematical relationships more than finding the answer.
- Without an over-emphasis on the answer, there were seemed to be no students that seemed to be lacking perseverance.

Hypothesis 5 – Moving the dialogue ball between students

Moving the dialogue ball amongst the kids because it is the group's responsibility to vet other student's ideas, examine them and decide whether to accept them into the community.

- Even though the teacher kept encouraging students to talk to each other, turn and face each other and discuss what mathematicians do, some students still want to talk to the teacher.

Individual team-member take-aways

- I like the idea of trying to use the community to vet students' ideas to decide what to accept as truth.
- I want to get better at modeling students' thinking and supporting students to articulate mathematical ideas.
- What the students are doing within their pairs during an investigation is part of developing an argument (argumentation is more than just the product, but rather also about the process).
- I need to be more comfortable with understanding the landscape so I can get underneath the thinking so I know what to move them towards. The landscape should help in moving away from focusing on how to get the answer.

Building a Mathematical Community Lesson Study

September 18, 2019

Research Question:

How can teachers support students to see that being in a mathematical community involves contributing to conversations about disciplinary ideas, to build on others' ideas and have others build on theirs?

The lesson study team hypothesized that the following actions would be important elements in building a mathematical community to enact over a period of months. Each hypothesis is listed below followed by the team's reflection.

Hypothesis 1 – Making implicit messages explicit

Implicit messages must be dealt with explicitly if we hope to improve the positioning of all students relative to mathematics (pointing out what mathematicians do, homework, grades, assessments)

- The teacher summarized some of what students are doing relative to what mathematicians do.
- Pointing out what was gained in the students' interaction validated some of the students to continue moving forward even if it was not their original idea.
- Several students made comments like "I learned there are many different ways to do the same question", "Always say what you have because it might help someone else understand", "Tell your people in your group what you are confused about", "Listen to what they're saying because…", etc.

- It's important that this is a recurring action that happens over time.
- The teacher can re-voice and extend students' thinking and simultaneously point out that what students are doing is part of doing mathematics.

Hypothesis 2 – Creating structures to position students as mathematicians

Establishing norms for participation involves creating structures to position each and every student as a full participant in mathematics and recognizing that participation builds agency (Turner et al. 2013).

- Many students seemed to be actively listening and actively participating in the discussions.
- The teacher can re-direct students' questions back towards asking other students those same questions, possibly using the sentence stems.
- The sentence stems seemed to communicate the type of discussion that is expected to be had.
- The teacher allowed students to compare representations or ways of thinking and decide which of those ideas should be displayed publicly in the whole group discussion, shifting some authority to the students.

Hypothesis 3 – Tasks that require "doing" and thinking

Requiring both of these types of processes (doing and thinking) has the potential to position students as doers and thinkers while simultaneously positioning mathematics as a discipline in which actions are coupled with reasons and justifications.

- When students were having small and whole group discussions, they were saying what they were doing and why, which caused them to explain their thinking.
- Students were trying to make sense of what they did through thinking and talking about what they did.
- The mix of conceptual understanding and procedural fluency allowed for students to have discourse that integrates their ideas.

Hypothesis 4 – Connecting conceptual understanding to procedural fluency

Connect conceptual understanding with procedural fluency to reduce mathematical anxiety and position students as mathematical knowers and doers.

- Students had difficulty trusting their own ideas in this lesson.
- Many students seemed to value getting the right answer over anything else.
- Some students were trying to connect conceptual ideas to procedures through drawing a model.

Hypothesis 5 – Explicitly elevating students' status

Make student thinking public, and then choose to elevate a student to a more prominent position in the discussion by identifying his or her idea as worth exploring, to cultivate a positive mathematical identity.

- A student with low status in the room was asked to write her equation on the board during the whole group discussion.
- The teacher repeatedly recognized which student's brought forth helpful ideas during the whole group discussion and was explicit about what was helpful about them.
- The teacher intentionally asked new students to publicize their ideas in the whole group discussion.

Hypothesis 6 – Sentence stems

A technique for shaping classroom discourse productively is the use of "sentence stems" aimed at promoting accountable talk.

- Many students seemed to be engaging in accountable discourse at their tables without using the sentence frames.
- Some groups made negative remarks about using the sentence stems.
- Other students did use the sentence stems in a way that may have helped to keep their discussion productive.
- Some students use the sentence stems in a somewhat resistant way, but nonetheless used them in their discussion.
- The team is wondering whether just using one or two sentence stems is better (or more accessible to students) in the future.

Recognizing that building a mathematical community does not happen in one lesson, each member of the team committed to working on these 3 hypotheses between this lesson study cycle and the next.

#1 Implicit messages must be dealt with explicitly if we hope to improve the positioning of all students relative to mathematics (pointing out what mathematicians do, homework, grades, asessments)

#5 Make student thinking public, and then choose to elevate a student to a more prominent position in the discussion by identifying his or her idea as worth exploring, to cultivate a positive mathematical identity.

#6 A technique for shaping classroom discourse productively is the use of "sentence stems" aimed at promoting accountable talk.

Doubles/Near-Doubles Lesson Study

September 20, 2019

Research Question:

How can teachers facilitate and/or support meaningful, student-centered discourse while posing purposeful questions that advance all students' reasoning?

The lesson study team hypothesized that the following actions would be important elements in supporting meaningful, student-centered discourse while posing purposeful questions that advance all students' reasoning. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Celebrate, get underneath and challenge students' thinking

Questioning that advances students' reasoning involves the teacher considering what the children are trying to do, celebrating and getting underneath what they've done or are trying to do, then challenging.

- For students that were having trouble representing the situation, keeping the students grounded in the context and allowing them to self-correct.
- The teacher made significant effort to make sense of what they students' were trying to do during the investigation.
- There was a lot of teacher re-voicing students' ideas in the investigation that helped to get underneath students' thinking.

- This structure of questioning can also be used in a whole group setting (celebrating, getting underneath and challenging)
- This also seems to be a time to support students in the development of more formal language (even, odd, etc).

Hypothesis 2 – Using reliable trajectories for questioning

Teachers' deep understanding and use of a reliable learning trajectory is important for posing purposeful questions to advance students' reasoning.

- The more we understand the landscape, the better we will become at asking questions.
- The standards can be seen as year-end outcomes, but are not enough to drive decisions about what questions to ask.
- There is a distinction between strategies and big ideas, and when strategy is going to become a big idea. Knowing how to use the right model to allow those big ideas and strategy to come out.
- The landscape helps the teacher decide when to push on big ideas that are on the horizon.
- The landscape should be useful in helping to figure out where students are and adjusting questions based on the information that we collect.

Hypothesis 3 – Considering the perspective of their audience

Having students consider what they are trying to convince their audience of before going to the gallery walk/congress helps students to consider the perspective of their audience.

- The teacher kept asking the students to defend their answer, or explain to "us" instead of "me".
- Over time, asking students to speak to the audience of other students should habituate them towards discussing with each other instead of just with the teacher.
- Explaining thinking and logic is an important step in developing an argument for an audience.

Hypothesis 4 – Re-voicing, asking students to re-voice each other

Using the teacher discourse moves during the congress may keep the discussion more student-centered.

- There were numerous opportunities for students to turn and talk when students brought up big ideas in the whole group discussion.
- There were moments where the teacher used "wait time 2" to allow students to build on each others' thinking.
- Students were building on each others' ideas repeatedly during this congress.
- Students were given opportunities to build on each others' ideas through explicit direction to do so, but they also added on to each others' ideas without prompting.

- Modeling students' thinking during the congress may help to keep the congress focused on students so they can examine their own thinking and logic.

Individual team-member take-aways

- It seems like the landscape is central to purposeful questions, which would then support a meaningful congress and later to develop argumentation. (Kendra)
- I want to go back and explore big ideas that students bring up and ask for students to prove conjectures that students bring up. (Annette)
- Recognizing that it's ok to spend more time on the same task/investigation from the day before. (Tess)
- Some of the nuances of models student thinking on the rack, how many students to have discuss in the whole group discussion were helpful to see in this cycle. (Sam)
- The landscape should guide how we decide what we want to have discussed and brought out in the congress and support students to consider that their audience is each other. (Kristy)

Linear and Quadratic Relationships Lesson Study

September 24, 2019

Research Question:

"How can teachers support students in developing the use of mathematical models as tools for thinking so that they can be used to model and argue their own thinking and reasoning?"

The lesson study team hypothesized that the following actions would be important elements in supporting students to develop models as tools for thinking. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Using tasks that promote conjecturing and justifying

Students should work on cognitively demanding tasks, represent their solutions, come up with conjectures (at various times throughout the task), provide justification, be prepared to share their findings with the class and make connections between different strategies/models/solutions.

- The task was cognitively demanding and engaging. Students struggled productively throughout the task.
- Students were trying various ways to think about the problem, testing them out and revising their models and thinking.

- Students were trying to justify their thinking with each other in working on the task, not just trying to find an answer.
- The justification pressed students to produce models.
- Many students used "brute force" methods for working on the task.
- As students were revising their models or thinking, they were trying to make connections across their models.

Hypothesis 2 – Making connections between models

The teacher should encourage students to make connections between different solutions and models, find the relationships between those models (conjecturing about when/why a model might be useful) to support students in using the more sophisticated models in future instances.

- In the whole group discussion, students were trying to connect between the area model and the equations.
- It seems that we implicitly asked students to make connections because of the questions that were asked.
- Some students were trying to represent the pattern with equations without being prompted to do so.

Hypothesis 3 – Providing opportunities to revise models

Representations should be developed in iterative cycles where students are given opportunities to revise their models.

- Many students made multiple revisions in their models.
- Students worked on developing/revising models for 30 minutes.
- The process students were using, building the model, checking the model to see if it was working and then revising the model occurred on many occasions in this lesson.
- Teachers' questions to advance students' reasoning while they are on the investigation before the whole group discussion can be used to promote revision of their models.
- The task of asking students to convince the rest of their class might encourage students to produce models that are revised throughout working on the task.

Hypothesis 4 – Modeling students' thinking over time

When students see their teachers and other students using models to represent their own thinking over and over students will begin to transition to using models for thinking.

- Many students were talking about and using tables and using ratios to model some of the relationships in the pattern.
- Many students also used area models to represent the situation and think with.
- Students were using models as tools for thinking in view of other students, which might support students to use models in the future.

- In the string, it seemed that most of the modeling of student thinking occurred on the table and students used tables more to think with than the double number line.
- The double number line might support more students in thinking about mathematical ideas and supporting number sense.
- A table may show scale factor more clearly and support students better who have strong number sense.

Hypothesis 5 - Modeling students' thinking during strings of related problems

Modeling students' thinking during a string of related problems will support students to use models as tools for thinking.

- It seemed that students were more likely to use tables in future events because the teacher was modeling students' thinking on a table during the string.
- Students used the table as a model of students' strategies.
- Students were certainly using tables as tools for thinking as described in the research. They chose to use tables, without prompting from the teacher, built the table to think with as they solved the problem.

Individual team-member take-aways

- I want to plan ahead what student responses might be so I can choose appropriate models that will highlight the most important relationships. (Jennifer)
- I want to plan more efficiently to connect the big ideas to the current task or mathematics and consider what models might be more effective in moving towards those. (Jody)
- I need to learn more about how to support development specifically for the coordinate plane so it becomes used as a tool for thinking. (Ryan)
- I want to figure out how to get kids exposed to models so that it becomes automatic just to use them. (Kari)

Compensation Lesson Study

September 30, 2019

Research Question:

How can teachers pose purposeful questions to advance students' reasoning and develop number sense?

The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to advance students' reasoning. Each hypothesis is listed below followed by the team's reflection.

Hypothesis 1 – Celebrating, getting underneath and challenging students

The conferral begins with a discussion of the learner's ideas, a celebration of the interesting and important ideas inherent in their approach, and then "upping of the ante".

- In the congress, the teacher was asking students if they heard what another student just said.
- During the conferrals, the teacher was celebrating when students were modeling the situation.
- Sometimes when the teacher left the students with a challenge question, some students did not seem very motivated to continue working on the challenge question.
- Some students did advance their reasoning as a result of the conferrals during the investigation.

Hypothesis 2 – Re-voicing students' thinking to formalize ideas

Getting underneath students' strategies could include re-wording ("are you saying that…") and making their ideas slightly more formal or including introducing new symbols/equations, etc.

- The teacher was re-voicing students during conferrals.
- Re-voicing students' ideas was common during the congress.
- Re-voicing students' thinking seemed to fit with modeling their thinking on equations.
- In one conferral, the teacher used a student's idea in a T-chart that the student extended when the teacher walked away.
- During the congress, students began generalizing about addition using the equivalent expressions to introduce the terms addends and sum.

Hypothesis 3 – Advancing reasoning through generalization

Advancing reasoning questions could include examining structure and regularity, finding ways to make the strategy more efficient, examining the problem with different models as tools for thinking, generalizing beyond the problem or writing an argument to convince others.

- The teacher was pushing students to generalize their ideas in the congress, asking if they thought that it (compensation and commutativity) would always work.
- The teacher can model a student's thinking (in this case on the T-chart) to support students' likelihood to examining structure and regularity that is inherent in their work.
- There were several instances where the teacher asked students to generalize or consider whether their thinking would extend to other situations: How do you know if you found all the ways? Would this always work?
- Students who were struggling to model the situation seemed to be able to make progress when being asked to connect to the context (bunk beds).

<u>Individual team member take-aways</u>

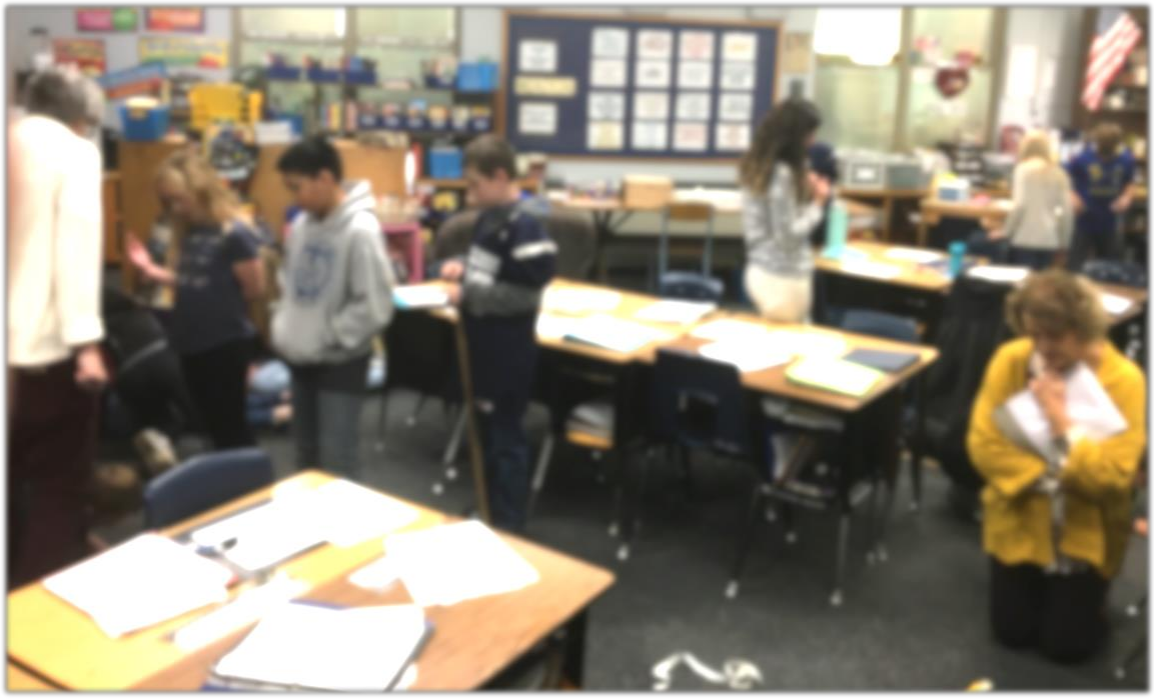
- I would like to get more familiar with the landscape and use it in questioning to know where to go with kids who are ready to move on the landscape as well as use it for differentiation. (Lisa)
- I'd like to find ways to pose purposeful questions with other curricular materials. (Lisa)
- I'd like to find tasks that students will have opportunities to struggle with productively that will allow me to pose questions that advance students' reasoning. (Melissa)
- I would like to work on celebrating, getting underneath and challenging during the students' investigations. (Edna)

Decimal and Place Value Patterns Lesson Study

October 8, 2019

Research Theme:

Supporting students to develop and defend convincing arguments while attending to precision



The lesson study team hypothesized that the following actions would be important elements in supporting students to develop and defend convincing arguments while attending to precision. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Shifting attention towards relationships

The teacher can use questions that shift the focus from computation with numbers to noticing and articulating the properties and relationships of these numbers.

- During the investigation, the teacher asked a group of student what 1/10 meant to expose the relationship between 1/10th of and dividing by 10.
- During the congress, it seemed important for students to have informal discussion about relationships that did not include numbers.
- The culture of the classroom clearly enabled students to be willing to to share their ideas and build on each other's ideas.

- Switching the communication context from whole group to small group enabled students to develop ideas in the small group setting, as well as build conjectures, claims and generalizations.
- Questions like….can you find a pattern, can you use another students' idea to figure out something else,
- Re-voicing students' ideas or having students re-voice each other seemed instrumental in structuring the discussion around the most important ideas.
- Warning students that the teacher is going to walk away after asking an advancing question might be useful in supporting productive struggle.

Hypothesis 2 – Building arguments collectively

Students should be collectively building arguments as a team as a way for students to understand that they can respect each other for the innate abilities that they have (community-building). The unpacking and development of ideas is often absent in students' attempts at argumentation and proof.

- Many students were respectfully agreeing/disagreeing with each other.
- Many students were building on each other's ideas in the whole group discussion.
- The use of various numbers in the chart that were sequenced through various students' ways of thinking supported students to unpack the ideas that the digits shift to the left when multiplying by ten and shift to the right when multiplying by 1/10.
- Students in this class seemed habituated to re-voicing each other, and did so without prompting from the teacher which then allowed more opportunities to unpack mathematical ideas and build arguments.

Hypothesis 3 – Using flawed argumentation

Teachers should design lessons to allow flawed argumentation to surface for discussion and intentionally focus on them.

- There were several instances where discussions occurred around flawed arguments in the small group work.
- The teacher re-voicing students' ideas could lead to students discussing these flawed arguments.
- In the whole group discussion, some imprecise or flawed arguments were welcomed for discussion.

<u>Individual team-member take-aways</u>

- I want to think more about the nature of the questions that shift students' attention away from computation towards general relationships when all students are using a common task. (All of us)

5th Grade Fractions Lesson Study

October 21, 2019

Research Question:

How can teachers use students' thinking and questioning strategies to support students in advancing their reasoning and using increasingly more efficient strategies (without taking over their thinking)?

The lesson study team hypothesized that the following actions would be important elements in using students' thinking and questioning strategies to support students in advancing their reasoning and using increasingly more efficient strategies without taking over their thinking. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Using questions to differentiate instruction

Teachers can use purposeful questioning to differentiate instruction for students based on the way they choose to start thinking about the task/investigation so each student needs different advancing reasoning questions.

- The teacher was asking different questions based on what the students had already produced, with many "how do you know" and "how do you know that's true" questions.
- The teacher was also asking, "If you know ___, how does it help you solve the problem?"
- Students that reached an impasse, the teacher asked if they thought they could produce a model which helped students move forward.

- The teacher was asking students to write some of what they were saying in mathematical symbols.
- The teacher used many of the questions that the team planned, including asking if the students were representing/modeling the situation.
- The teacher was pushing students to relate their thinking to the model that was connected to the context.

Hypothesis 2 – Celebrate, get underneath and challenge

During an investigation, the teacher should assess students' reasoning first, celebrate what they're trying to do, and ask advancing reasoning questions that either make mathematics visible and/or encourage reflection/justification.

- Part of these discussions included time where they teacher was asking students to clarify their current thinking.
- The teacher used question type #5 (engaging with the reasoning of others) during the investigation, asking the partner what they think and if they agree/disagree. This can be used the try to gain consensus within the group.
- The pairs of students during the investigation seemed to be optimally mismatched which allowed for strong discourse between the students.
- How did you know that you ____ ? probes thinking and encourages reflection and justification, but seems to need more follow-up questioning to advance students' reasoning.
- Modeling students' thinking seems important for the students to be able to see their own thinking in a slightly more formal way.

Hypothesis 3 – Questions to support building on each other's ideas

Focusing pattern of questioning in whole group discussions involves listening carefully to students' ideas, pressing them to communicate their thinking, and encouraging students to build on each other's ideas. (Question type 5 – engaging with the reasoning of others)

- Strategically selecting students' models or representations tied to the big mathematical ideas we want students to have discussion on is important for students to be able to build on each other's ideas/thinking.
- It seems appropriate and important for the teacher to model students' thinking for the whole community to make connections to, then giving students the opportunity to try to connect their own thinking to that model.
- The teacher was asking students to explain other students' models which allowed them to build on each other's ideas.
- The teacher was also asking students to make connections between the various models and ways of thinking that also allowed them to build on each other's thinking.

Hypothesis 4 – Wait time one and two

Using wait time one and wait time two will allow more students to volunteer to respond, the number of "I don't know" responses decrease, the length of students' responses increase, and students react more to each other's statements.

- Although the teacher may not have waited for 3 seconds, students did add on to their own thinking when given a second after they said "I don't know."

Hypothesis 4 – Wait time one and two

Teachers can frame the discussion by encouraging students to include specific parts of their thinking in poster through their advancing reasoning questions.

- The teacher specifically told some students to share certain aspects of their thinking on their poster, which then became part of the whole group discussion.
- Some students explicitly showed equivalent expressions on their poster because the teacher encouraged them to.

Individual team member take-aways

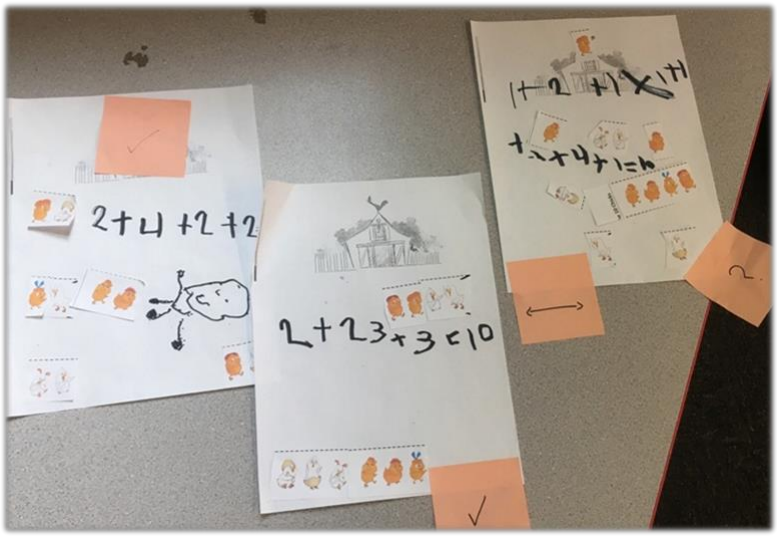
- I want to think about what the lesson is trying to accomplish, plan out how they will respond to the task, and how I'm going to respond to what they choose to do so it's not improv. (Jody)
- It was helpful to see what the assessing, celebrating, advancing reasoning questions looked like in action. (Sharon)
- I want to continue working on questions that advance students' reasoning without taking over their thinking. Questions need to be connected to students' thinking. (Leti)
- I realize that I need to trust my students that they can come up with important ideas. (Lydia)
- We need to remember that all students come in at different places and they won't all learn the same thing as other classmates' throughout the unit. (Sharon)

Kindergarten 5 and 10 Structures Lesson Study

October 22, 2019

Research Question:

How can teachers support students to develop and defend convincing arguments while building agency and authority in the primary grades?



The lesson study team hypothesized that the following actions would be important elements in supporting primary students to develop and defend convincing arguments while building agency and authority. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Refining mathematical language

Teachers can use re-voicing (modeling student thinking) and questions to support students in refining their mathematical language to develop language appropriate for argumentation within their community.

- The teacher asked students to model their own thinking during the investigation on the number rack from the context.
- During the string of related problems, the teacher would models students' thinking on the board in the form of equations.
- The teacher's questions seemed to promote the use of equations to match their thinking, particularly the question of "Is the gang all there?" and "Can you show it?" to connect their thinking to the total that they were working with. Thereafter, students were then writing equations to match their strategy/thinking.
- The recording of the equations during the congress, including equations with parentheses, seemed to allow students to notice the relationship between the other students' thinking and the 5-structure.

- Students were connecting their peers' thinking and the modeling on the rack to the context, calling the bead and numbers "chicks".

Hypothesis 2 – Using teacher discourse moves to focus the conversation

Teachers can use re-voicing, asking students to re-voice and wait time 1 and 2 to structure and focus the conversation on an important idea.

- During the congress, the teacher re-voiced students' ideas on multiple occasions and asked students to re-voice each other's ideas keeping more students in the conversation and allowing the conversation to be centered on important big ideas.
- The students seemed comfortable with wait time 1, which may come from doing lots of strings of related problems where they have to wait before they can share their thinking.
- Using turn and talks to ask students re-voice each other's ideas seems to be critical for keeping the whole group discussion moving in a productive direction, and allowing more students to share their ideas and contribute to the whole group discussion.

Hypothesis 3 – Interpreting student thinking with trajectories

Teachers can interpret student thinking in terms of reliable trajectories in order to shift away from a deficit mindset way of thinking.

- The team sees the landscape as a way for us to interpret student thinking in terms of what they do understand rather than finding "gaps" or holes in their thinking.
- The landscapes should help teachers to be more able to build on what they are thinking instead of trying to fix what they're not doing.

Hypothesis 4 – Habit of always questioning

Teachers can develop a habit of always questioning students, whether correct or incorrect, to help students learn to dig deeper into an idea and unpack concepts and develop the skills related to proving.

- Some of the questions in the investigation asking students to connect their thinking to the rack and the context to prioritize the 5-structure through the strategies they chose to use.
- During the congress, there were opportunities for students to dig deeper into what the expressions meant relative to the context and the rack, and then being pressed to determine of all the chicks are still there.
- During the congress, asking students to talk about what a student meant by "they're the same but different…." allowed students to unpack the idea of equivalence.

Other observations

- The gallery walk seemed to help students recognize that the other students are their audience.
- The gallery walk also stimulated discussion between students about their work, making suggestions for how they can improve their work.

- Students were asking thoughtful questions about each other's thinking during the gallery walk.

Individual team-member take-aways

- I am excited to use the videos for program to see how the staff can support kids in pull-out math and to support kids in the classrooms where kids are in math workshop, and to help focus on questions to ask to advances students reasoning. I also want to use number strings with my students. (Ashley)
- I will continue to learn how to manage some pieces with sticky notes during the gallery walks and using strings to focus on prioritizing the 5-structure. (Jordan)
- I want to learn how to use the landscape more and moving each student forward, focusing on differentiating through the lens of the landscape while also moving the whole class forward. (Melanie)

4th Grade Measurement Lesson Study

October 30, 2019

Research Question:

How can teachers support students to develop conjectures, investigate why they work so they can construct convincing arguments and justify their reasoning to other students during the math congress?

The lesson study team hypothesized that the following actions would be important elements in supporting students to develop conjectures, investigate why they work so they can construct convincing arguments and justify their reasoning to other students during the math congress. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Questions to shift students' attention away from computation

In conferrals, teachers questions should intentionally shift students' attention away from computation towards general relationships or the extending their reasoning beyond the range in which it originated by:

a. *having students investigate why their conjecture works or is true involves teachers getting underneath the mathematics will support students in attending to particular features and suppressing others.*
b. *model their thinking and the processes they use so they have objects to discuss and can examine their logic OR*
c. *suggesting they try to use a model that would convince others why.*

- Many of the questions the team planned and the teacher asked did shift students' attention away from the computation towards general relationships, but did not alone cause students to write about them in the arguments.
- When modeling students' thinking, it's important for the teacher to ask if it represents the students' thinking.
- Modeling of students' thinking can happen during the conferrals and the congress so that students will later use those models as tools for thinking.
- Students use models of thinking for their arguments after seeing their thinking modeled on that model over time.
- Re-voicing students' ideas is another vehicle for shifting students' attention away from computation towards general relationships.

Hypothesis 2 – Framing the conferral towards a conjecture to prove

Teachers should leave students in conferrals with the teacher writing their conjectures on a post-it note to articulate what they are trying to prove or convince the rest of the class of.

- This action may leave groups with residue of the conferral for students to re-visit as the construct their arguments/posters.
- Once it was written through the questions in the conferral, students had a focus for what to write on their posters.
- Some students who did not confer with the teacher simply wrote what they did on the poster, which made this team think that the conferrals were powerful in shifting students' attention away from the computation.

Hypothesis 3 – Articulating mathematical ideas prior to the congress

Students should be able to articulate the mathematical ideas that they discovered prior to deciding what they will be bringing to the community in the congress.

- The teacher should be trying to re-voice what he/she thinks the students know rather than telling them what they know.

Hypothesis 4 – Re-examine, revise and simplify

Students can re-examine, revise and simplify their ideas prior to the congress by sharing their ideas with another group to allow for revision of their written arguments. (Stronger and Clearer Each Time?)

- While we did not get a chance to do this in today's lesson, one member of the group has been doing this with their students on most days and finds it important for critiquing each others' thinking and revising their own arguments.
- The team wonders if the Instructional Routine of Stronger and Clearer Each Time would be useful in providing a structure for sharing and giving feedback.

Dividing Polynomials Lesson Study

November 5, 2019

Research Question:

How can teachers use the Mathematics Teaching Practices to simultaneously support students' view of themselves as thinkers and "do-ers" of math and see the value of and engage in progressive formalization of reasoning and argumentation?

The lesson study team hypothesized that the following actions would be important elements in using the Mathematics Teaching Practices to simultaneously support students' view of themselves as thinkers and "do-ers" of math and see the value of and engage in progressive formalization of reasoning and argumentation.

Hypothesis 1 – Teacher discourse moves

Using the teacher discourse moves of waiting, probing thinking, re-voicing and asking students to re-voice each other.

Observations

- The teacher asked students to re-voice each other in multiple occasions.
- Wait Time 2 occurred during the notice/wonder and a student added on to his own thinking without prompting.
- The notice/wonder was useful in allowing students to bring out their own ideas, which presented opportunities for them to re-voice each other.
- The teacher asked some students to share their thinking in the whole group prior to the discussion (warm-calling).

Implications for Teaching

- We could all wait longer after we ask questions, and after students are responding.

- Re-voicing is not difficult, but effective.
- Giving students an opportunity to re-voice each other provides greater access to mathematical ideas that did not understand the students' reasoning the first time the statements are made public.
- Having students re-voice each other also seems to be useful in helping students to see the value in each other's reasons.
- Probing thinking questions allow students' thinking to be made visible for other students to build on.

Hypothesis 2 – Questions to shift towards general relationships

Questions should shift students' attention away from the computation towards general relationships and properties.

Observations

- In the whole group discussion, the main question students were discussing was "Which expression is most useful?" which seemed to allow students to talk more generally about the difference between the different forms.
- The language students were using was more generalized comparing the usefulness of different forms of expressions.
- When students were asked what they notice/wonder in the beginning of the lesson, they were able to discuss less about computation and more about general properties of the representations and how the representations were connected.
- When students were struggling, the teacher asked the students to connect the representations of expressions and the diagrams to which seemed to allow students to move forward with the computation.
- Asking students to sketch a graph and describe why they did what they did promoted generalization of the relationships between the expressions, factors and graph.

Implications for Teaching

- Asking students "why" pushes them towards seeing themselves as thinkers and do-ers of math.
- These types of questions press students to use more formal or precise language because they are trying to communicate their ideas to others.
- Students need to be given enough time to think about the questions that would shift students' attention away from the computation towards general relationships.
- Teachers can prioritize time students get to think about questions that push for generalization by, at times, reducing the amount of time they spend on the computation by using paired discussions during the computation.

Hypothesis 3 – Articulating mathematical ideas prior to the congress

Careful questioning to support students in operating on and unpacking their own ideas and listening to students responses helps to build important aspects of a culture of argumentation.

Observations

- Wait time 2 seems to be an important component in unpacking students' ideas.
- Asking students to refer to what they were thinking during the notice/wonder seemed to help unpack their ideas.
- Asking students to re-voice each other also supports students in listening to and building on each others' ideas.
- The routines seem to provide the structures for students to unpack their ideas.

Implications for Teaching

- The routines allow students to engage in the mathematics and unpack their own ideas.
- The more unpacking of their ideas that students do, the better they should become at developing arguments because the students feel like what they are doing is important.
- Students should have regular opportunities to simply share their ideas without evaluation so that they will become increasingly more willing to engage in meaningful discourse.

Hypothesis 4 – Re-examine, revise and simplify

Listening and critiquing arguments supports students in learning new argumentation techniques and considering ideas other than their own.

Observations

- The Stronger and Clearer Each Time routine seemed to support students in asking questions of each other.
- The notice/wonder task paired with the teacher discourse moves provided opportunities for the students to critique each others' reasoning.
- During the "four corners" portion where students were to try to convince others which expression is most useful allowed students to collectively build arguments.
- Faulty reasoning is useful in helping students to revise their arguments.
- During the Stronger and Clearer Each Time routine, students considered faulty reasoning to be accurate until they changed partners where there was another opportunity to revise their. The second pairing was useful for giving students more ideas to revise their initial draft.
- Many students revised their thinking in the Stronger and Clearer Each Time routine.
- It seemed that students were willing to use the sentence frames and trying their best to use them, but it might need to be used more consistently in order for it to become more frequently used.
- The conversations in the first round strengthened the conversations and feedback in the second round.

Implications for Teaching

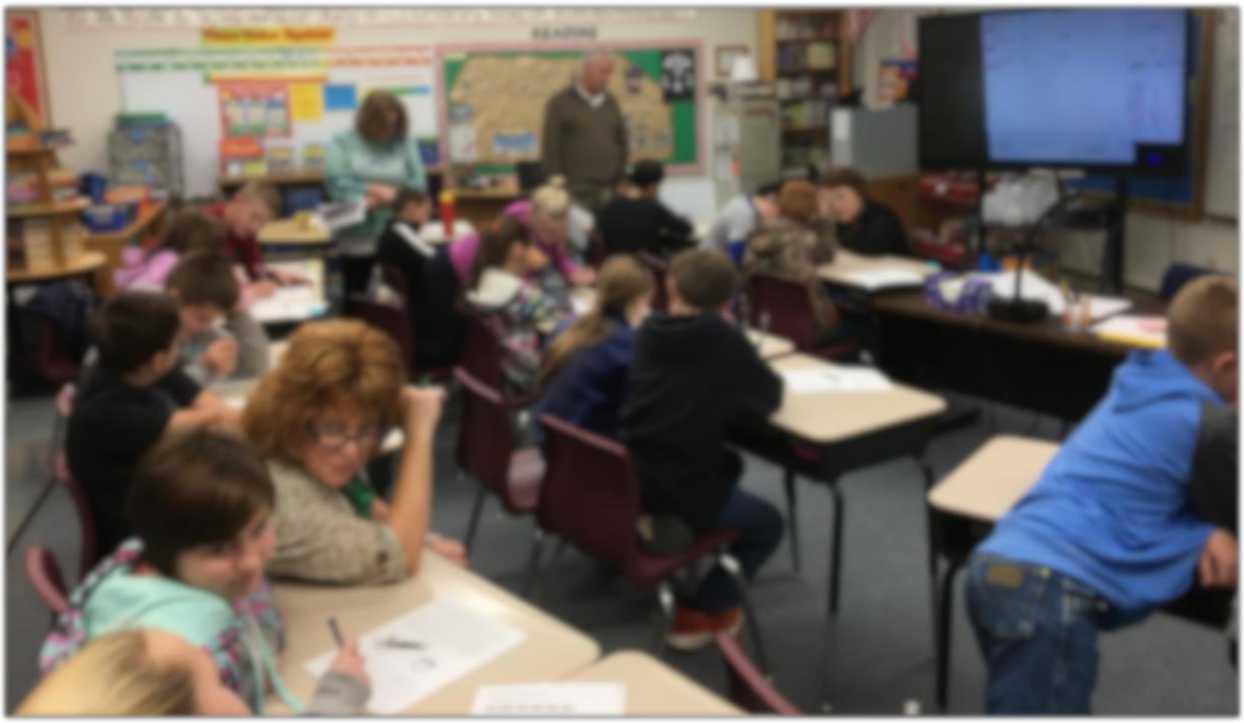
- This seems to support students in seeing that they are part of a mathematical community and that they can all learn from each other.
- We might expect students to ask more questions of each other for the benefit of everybody.
- Students should learn through this routine that feedback could come in the form of questions.
- Some students did want to trade papers during this routine, but many did not.

4th Grade Measurement Lesson Study

November 12, 2019

Research Question:

How can teachers use conferrals to support whole group discussion involving argumentation? How do we support students in moving from their own strategy towards generalization?



The lesson study team hypothesized that the following actions would be important elements in supporting generalizing and argumentation. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Questions to shift students' attention away from computation

Teacher's questions should shift students' attention away from computation towards general relationships to support students' noticing and articulating of those relationships, which paves the way towards building an argument.

- On multiple occasions the teacher asked students if they thought this idea would work every time.
- The teacher was not focusing on their answers, but more about the efficiency of their strategies.
- The teacher regularly asked students for patterns they are noticing in their work, and if they saw additional patterns beyond the first one that they noticed.

- When students were saying what they did, the teacher would ask students if they could see it differently to shift attention from skip-counting to the relationship between the units.
- The teacher regularly was pressing students to look for relationships within their table and was less focused on "filling out the table".
- Strings of related problems are also a structure that goes nicely with these types of questions because they promote "if__, then __" statements by the students.
- Students were using relationships within the table that other students were articulating that enable them to use strategies that they hadn't used earlier.

Hypothesis 2 – Re-voicing and having students re-voice each other

Teachers re-voicing of students' ideas (and asking students to re-voice each other) emphasizes that students' ideas are important and to structure and focus the conversation on an important idea.

- There were dozens of times when the teacher would re-voice students' ideas.
- The teacher was able to use re-voicing to refine mathematical language.
- Using re-voicing of students' ideas to make their claims more general helped students to write generalizable arguments.
- Re-voicing can also come in the form of celebrating what the student already knows and is a powerful way to start a conferral that supports productive struggle.
- Re-voicing also implicitly tells students that their ideas are important.
- This practice could also be important for encouraging students to stay engaged in the task.

Hypothesis 3 – Wait Time 1 and Wait Time 2

Wait time and slowing down fits well with efforts to develop argumentation and thoughtful mathematics.

- Wait time was evident during the string when students were thinking about their solution to each problem.
- Students may have been willing to share their reasoning because of the wait time during the string of related problems.

Kindergarten "Packs of Five" Lesson Study

November 13, 2019

Research Theme:

Supporting reasoning and argumentation

The lesson study team hypothesized that the following actions would be important elements in supporting primary students in reasoning and argumentation. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Conferring with students

Conferrals that support reasoning and argumentation should start by asking students if we can join their conversation, listening intently to their ideas, followed by celebrating and getting underneath students' thinking supports students to have ownership of their own strategies.

- There were many instances where the teacher began the conferrals by asking students if she could join the conversation then celebrating what they are trying to do.
- It seems important to ask students to join the conversation because students may/may not be ready to discuss their ideas at that moment.
- Listening authentically to what students are trying to do is critical to determine where they are working on the landscape.

- This structure seems to support students' willingness to take a risk in problem-solving and to support productive struggle (SMP 1).
- Re-voicing students' ideas seems to be an effective way of getting underneath their strategies/thinking.

Hypothesis 2 – Re-examining and revising ideas

Children need multiple experiences to re-examine, revise and simplify (generalizing) their ideas if they are to build a foundation for understanding a mathematician's view of proof (sequence of the tasks).

- The sequence of the tasks should allow for common, but slightly different experiences that promote movement on the landscape/trajectory.
- Referring to previously constructed models of thinking might be useful in re-examining/revising ideas.
- During the congress, the students were able to re-examine their strategy of counting by ones and have discussion about when that strategy might be more or less efficient.

Hypothesis 3 – Seeing the audience as the other students

Questions might need to support students in shifting their attention towards their audience (the other students).

- During the congress, the teacher was pressing students to think about what the other students were saying, asking them to turn and talk about other students' ideas.
- The teacher had asked one of the students during the conferral that presented during the congress if she would be willing to share her idea with the other students.

Hypothesis 4 – Shifting attention away from computation towards general relationships

Some questions should shift students' attention away from computation towards general relationships to support students noticing and articulating of mathematical properties and relationships.

- Focusing on the big ideas in the congress helps to stay focused on facilitating the conversation in a productive direction.
- One way to decide on the big ideas to ask our questions around in the congress is to identify first what they are doing on the landscape and trying to decide what might be next.
- When students are making errors, it seems to be useful to generalize what they are doing or trying to do to focus attention on the general relationships/strategies.
- During the congress, the students shared their strategy of what they did. The teacher asked if that is an efficient strategy, and in the turn and talk, one student made an if __, then __ statement.

3rd Grade Partial Products Lesson Study

November 25, 2019

Research Question:

How can teachers use tasks that allow all students to reason and problem-solve and pose purposeful questions to advance students' reasoning towards more efficient strategies?

The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to advance students' reasoning. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Conferrals to celebrate, clarify then advance reasoning

Conversations should start with celebrating their strategies or what students are trying to do, then re-stating or clarifying ("are you saying") and advancing their reasoning.

- The teacher was getting underneath students thinking by saying "I like how you grouped these to make a bigger chunk, and it looks like you doubled these."
- Students that were using repeated addition were celebrated by saying "I like that you knew to add $1.25 24 times."
- The teacher celebrated students' perseverance in trying other strategies when they did not have a procedure to use.
- Getting underneath and challenging included "that's a lot of adding though, is there another way we can group these".

- This structure is important to help students see that they are inventing the strategies and owning their explanations and justifications.

Hypothesis 2 – Convincing others and generalizing

Students should be asked to try to convince others that their reasoning is sound, or that their strategy will work in other situations.

- The teacher was asking students to discuss their ideas with each other.
- Will you tell them why it worked for you?
- Explain this part of your thinking to ___.
- The teacher was asking students to see where their thinking was in their partner's strategy, but some students had trouble doing so.
- Explain to your partner why you did ____.

Hypothesis 3 – Using questions to facilitate student-student discourse

Questions should be used to support students in discussing their ideas with each other.

- During the congress, students were asked to interpret each other's thinking.
- It seemed useful to ask students to explain why they think the student presenting used a certain strategy.
- Can you share what you did with your 25 cents?
- Modeling of students' thinking seems to be a productive strategy for helping to move the community forward.
- Turn and talks are an important strategy to keep the discussion moving between the students.
- The team felt like most or all students were able to see some new mathematics coming out of the congress relative to what they were thinking going into the congress.

Supporting Use of Mathematical Models Lesson Study

December 9, 2019

Research Question:

How can teachers support students in developing the use of mathematical models as tools for thinking and deeply understand them so that they can be used to stimulate meaningful discourse?

The lesson study team hypothesized that the following actions would be important elements in supporting the development of mathematical models as tools for thinking. Each hypothesis is listed below followed by the team's reflection.

Hypothesis 1 – Teacher modeling students' thinking

When students see their teachers using models to represent their thinking over and over again and when they are allowed to investigate rich contexts where these models arise naturally, they transition from models of thinking to models for thinking.

- During the string, students were making general statements about the inverse relationship between the 2 variables.
- The teacher was modeling students' thinking during the problem string that some students were using to work on other pieces of the table.
- Students used similar modeling on the table that the teacher used during the string in the task that followed the teacher's modeling of students' thinking from the problem string.
- The teacher encouraged several students to make a different model, create a table or graph to look for patterns.

- It seems that the ongoing, over and over action of modeling students' thinking is likely to support students in using models for thinking.

Hypothesis 2 – Progression of the development of mathematical models

The progression of models and modeling includes:

1. *Models of a situation*
2. *Models of students' thinking*
3. *Models as tools for thinking*

Hypothesis 3 – Shifting thinking towards relationships

By asking questions to shift students' attention towards relationships between the models, students' construct new mathematical objects and engage in iterative cycles of modeling.

- The teacher regularly asked questions about what's happening in their work that pressed them to think about or discuss relationships between the variables.
- The teacher was asking questions to support students in modeling the situation.
- The teacher was asking students to look for patterns in their tables that will, over time, support students in seeing relationships like rate of change.

Hypothesis 4 – Withholding our authoritative stamp of approval

We need to withhold our authoritative stamp of approval and give the mathematics back to the students to reflect on and sort out.

- Using wait time 1 and wait time 2 could be useful as a strategy for withholding the teacher having the authoritative stamp.
- Strings of related problems might also be a useful structure for allowing students to have authority over the mathematics.
- The teacher consistently withheld her authoritative stamp of approval in this lesson and students were still doing the reasoning and problem-solving.

Hypothesis 5 – Selecting/sequencing students' representations

Selecting and sequencing of representations during the whole group discussion should be used to advance the mathematical goals of the lesson.

- Most students chose not to use a graph as a model for thinking, so the teacher needed to encourage the students to choose a different model.
- Many students who were encouraged to use a graph to represent their thinking, but not as many used as a tool for thinking.

- Asking students about the relationships in the graphs might support students' development in using graphs as tools to think with.

Summary and key take-aways

- Provide rich contexts where models arise naturally.
- Encouraging students to use more than one model and/or identify connections between models.
- Highlight important relationships on models and where they are in various representations (rate of change in table and graph)
- Model students' thinking on a model we want them to use during strings of related problems or during investigations
- The deeper we understand the mathematics and development of mathematics from students' perspectives, the more powerful we are as questioners and facilitators of meaningful discourse.

4th Grade Mathematical Models Lesson Study

December 10, 2019

Research Question:

Supporting the development of mathematical models as tools for thinking

- *How can students' use of models support the advancement of strategies they choose to use?*
- *How can students' use of models move away from or go beyond just getting the answer?*

The lesson study team hypothesized that the following actions would be important elements in supporting students' use of mathematical models as tools for thinking. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Promoting certain models with contexts

The context will impact how students model the situation, and the model will then impact what relationships the students will be able to notice.

- The array model allowed students to see the relationship between partitive and quotative models of division.
- It is critical for teachers to understand which models to push because different models highlight different relationships.
- The model allowed students to see the mathematics and relationships better than had the model not been developed.

- Having the open array as a discussion piece allowed students to use more formal and precise language (students were referring to what the 6 represented in the different contexts).

Hypothesis 2 – Questions to support generalization across models

Questions should be used to ask students to generalize across contexts and models to support their development of models used as tools for thinking. (whole-group discussion)

- When asked whether they thought 156 divided by 6 represents the water or juice, many students who had not constructed that relationship prior to the whole group discussion.
- Students began to discuss the model more generally (columns versus flavors) after the students' thinking was modeled on the open array in the whole group discussion.

Hypothesis 3 – Modeling students' thinking

We need to model students' thinking and the processes they use (phase 2 of modeling)

- When the teacher modeled the students' thinking from the array to the open array we could see the students making connections between the quantities.
- The array model allowed students to see the relationship between partitive and quotative models of division.
- Asking students to produce another model seems insufficient in moving students along in development of a models used as a tool for thinking.

Hypothesis 4 – Wait time one and two

Progression of models/modeling

- Teachers should be aware of and leverage the progression of the development of mathematical models over the course of a lesson, unit, year and grade span.

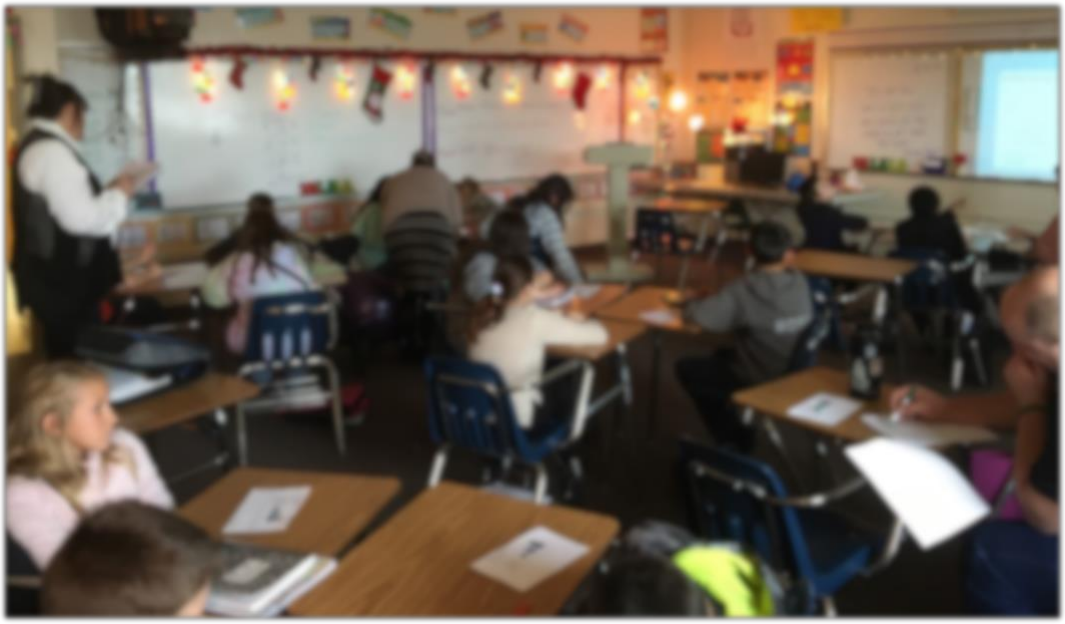
    1. Model of a situation
    2. Model of students' thinking (generalizable)
    3. Models for thinking

6th Grade Equivalent Ratios Lesson Study

December 11, 2019

Research Question:

Supporting students to develop and defend convincing arguments



The lesson study team hypothesized that the following actions would be important elements in supporting generalizing and argumentation. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Re-voicing to structure the conversation

If the teacher re-voices students, or asks students to re-voice each other's ideas, we can structure and focus the conversation on an important idea and potentially have students add on to each other's ideas.

- Re-voicing students' ideas seemed to allow at least one opportunity for students to build on other students' ideas.
- It seemed productive to have small group discussions between students when they are asked to revoice what another student said in the whole group discussion. This seemed to allow students to add on to each others' ideas.
- Re-voicing students' ideas seemed to be useful when switching the communication context from whole to small group.
- The small group discussions seemed to be focused on the important mathematical ideas because the teacher re-voiced students' ideas prior to asking them to talk in small groups.

Hypothesis 2 – Questions shifting attention away from computation

Questions should shift students' attention from computation to general relationships, and prompt for clarification, justification, explanation, elaboration and critique to support unpacking and development of ideas (building blocks of argument)

- Modeling students' thinking on generalizable mathematical models like the ratio table and double number line will support students in attending to general relationships.
- When students were asked why all these strategies work, students generalized about proportionality.
- Students were constructing new generalizations as a result of a few questions that they were allowed to talk about in small groups.
- Many students were also making connections between the double number line and the table.

Hypothesis 3 – Teacher discourse moves to position students

Using the teacher discourse moves can elevate the positioning of students with low socio-mathematical status.

- It did not seem obvious in this classroom who had more status than others.
- Wait time is helpful in providing students opportunities to elevate status and engage in reasoning.
- Using the discourse moves may allow the teacher to see students as more capable than previously.
- The teacher intentionally re-voiced students' ideas to emphasize important relationships, which simultaneously elevates students' positions.

Hypothesis 4 – Strategically changing the communication context

Strategically changing the communication context (small and whole group discussions) can be an important vehicle for the teacher to warm-call students to support positioning.

- Students are more likely to share partial arguments in the safety of a small group.
- Warm-calling can be useful when switching from CC1 to CC2.
- Students with low status seemed to be willing to contribute significantly when the teacher used warm-calling.
- Cold-calling is easier for the teacher to use because high status students will move the lesson quicker, but not necessarily with better results.
- Strategically changing the communication context elevated students of low status and also led to students' discussion of general relationships.
- Allowing students to share their ideas in the whole group and then using those ideas for everybody to discuss in small group multiplies the status change.

Kindergarten Argumentation Lesson Study

January 16, 2020

Research Question:

Supporting students to develop and defend convincing arguments

The lesson study team hypothesized that the following actions would be important elements in supporting students to develop and defend convincing arguments. Each hypothesis is listed below followed by the team's reflection.

Hypothesis 1 – Modeling students' thinking

Teachers need to model students' thinking and the processes they use so they have objects to discuss and can examine their logic.

Observations

- On the math rack, when students talked about commutativity, the teacher flipped the math rack to model their thinking.

- Modeling students' thinking forces the teacher to deeply understand what the students are doing.
- Modeling students' thinking is important in supporting students to inherit models as tools for thinking.
- Two models that students saw connections between allowed students various access points to mathematical reasoning and articulating relationships.
- The teacher consistently asked students to make connections between various representations/models and the context.

Implications for teaching beyond this lesson

- Consistent modeling of students' thinking will help students develop claims because they have an image or object that helps to articulate relationships and defend their thinking because they can refer to the model in their explanation.
- Modeling students' thinking could also support students in developing a structure for developing arguments in the future.

Hypothesis 2 – Noticing regularity, articulating and investigating claims

Students should be prompted to notice regularity, articulate a claim (allowing for revision), investigate the claim through representations to construct arguments to be considered as a whole group conjecture/argument.

Observations

- During the congress, we sequenced the student work and modeled their thinking on a consistent model (linking cubes and chart) to allow students to have an opportunity to notice regularity.
- The sequence of students' work strategically supported the development of potentially using a systematic production of arrangements in future tasks.
- The teacher asked students what the students noticed in the model and they were noticing regularity (one goes up while the other goes down, going up or down by one, predicting the next one in the sequence)
- The following task will have students generate combinations for other numbers to have an opportunity to generalize compensation.

Implications for teaching beyond this lesson

- Supporting the development of a systematic production of arrangements will help students notice key features.
- Questions and tasks should be intentionally supportive of helping students to notice the most important relationships for students to notice.

Hypothesis 3 – Using the congress to attend to each other's ideas

The congress can be used to support students in attending to each other's ideas.

Observations

- Many students were interested and building on each other's ideas in the congress.

- Many students were seeing the regularity and relationships in the models that they could connect to the context.
- Using their ideas seemed to provide buy-in to their discussion with each other.
- Having a visual representation of the task and their ideas likely kept students involved in the discussion.
- Re-voicing and asking students' to re-voice each other is helpful in productive lingering on important ideas that came out in the congress.

Implications for teaching beyond this lesson

- When students are building on each others' ideas, we may need to use the next day to investigate why their ideas are working on the following day.
- The congress allows the class culture to build in terms of learning to communicate with each other.
- There are also opportunities in the congress (and during the investigation) for students to learn to listen to each other.
- The congress is also a good structure to allow for productive lingering.

Generalizing Strategies Lesson Study

January 17, 2020

Research Question:

Supporting students to develop and defend convincing arguments

The lesson study team hypothesized that the following actions would be important elements in supporting students to develop and defend convincing arguments. Each hypothesis is listed below followed by the team's reflection.

Hypothesis 1 – Re-voicing and having students re-voice each other

The teacher can re-voice or ask students to re-voice each other to focus the conversation on an important idea or acknowledge a noticing.

Observations

- The teacher was re-voicing students' ideas during the string of related problems.
- The teacher was re-voicing students' ideas when they began to try to justify why the strategy works.
- Several students were asked to re-voice how other students were trying to connect the equations to the number line.

Implications for teaching beyond this lesson

- Re-voicing gives the students another opportunity to hear the ideas that should provide more opportunities to generalize or connect to something else.
- When the teacher re-voices, the students realize that their thoughts are important.
- Re-voicing allows students to crawl into the conceptions of other's ideas so that they can add to them.
- When students hear themselves re-voiced, they are more likely to contribute to the conversation because it raises their status or position in front of their peers.

Hypothesis 2 – Questions to shift attention towards general relationships

The teacher can use questions to invite noticings of relationships, properties and regularities of their own emerging ideas to move toward building arguments that are more general.

Observations

- The gallery walk was inviting noticings of relationships.
- In this lesson, the claim was already given to students but they had lots of opportunities to notice relationships and regularities that were moving in the direction of more general relationships.
- Students were asked in the congress why the claim was or was not true.
- Students were asked whether using examples is enough to prove why this strategy works.
- The routine of strings of related problems in itself should promote a culture of looking for general relationships.
- Asking students to connect the model to the equation, was helpful to start a discussion about general relationships.

Implications for teaching beyond this lesson

- Questions to shift towards general relationships might need to be more specific to narrow all the possible relationships they can look for to focus on certain parts of the situation.

Hypothesis 3 – Modeling of students' thinking

If we model students' thinking and the processes that they use they will have objects to discuss and can examine their logic.

Observations

- There were obvious examples of this action occurring during the number string.
- During the congress, students were using the model to investigate the claim.
- The teacher was modeling students' thinking on the double number line and with equivalent expressions during the investigation.
- By modeling students' thinking during the conferrals, students inherited these models to continue generating examples with those models.
- The modeling of the student's thinking was clearly connected to their own thinking.

Implications for teaching beyond this lesson

- Having a model in addition to thinking should help to deepen students' understanding of a big idea or strategy.
- This teaching practice might help to students bridge the data to the warrants and help students justify why their strategies work.
- Using this teaching practice might help teachers to deepen their own understanding of the mathematics.

Hypothesis 4 – Co-constructing a community argument

Teachers can support the co-construction of mathematical arguments through documenting students ideas during the congress in the form of claim, data and warrants.

Observations

- This format might help students to see that mathematics is not just about doing linear procedures, step by step.

Implications for teaching beyond this lesson

- This has potential to be a public document for other classrooms to examine and provide feedback.
- Publicizing argumentation in mathematics is a move away from answer-getting and defining argumentation as an important part of doing mathematics.
- This format may also help students to see that mathematics is also about justifying and clarifying ideas that others can look at and question/revise.

Supporting Productive Disposition Lesson Study

January 28, 2020

Research Question:

How can teachers use the Mathematics Teaching Practices to simultaneously support students' view of themselves as thinkers and "do-ers" of math and see the value of and engage in progressive formalization of reasoning and argumentation?

The lesson study team hypothesized that the following actions would be important elements in supporting students to view themselves as know-ers and do-ers of mathematics. Each hypothesis is listed below followed by the team's reflection.

Hypothesis 1 – Re-voicing and having students re-voice each other

Creating opportunities to engage with another's reasoning can be used to help students see connections between solutions that have been generated by students of differing statuses, which can provide legitimacy to many students' contributions.

Observations

- By allowing students to talk in small groups, students were much more willing to ask questions of each other and try to interpret each other's thinking.
- Students seemed motivated during the small group discussion to engage with other students' reasoning.
- Using students' representations to be displayed in whole group discussions could also be movitating for students to continue problem-solving through the task.
- Students were encouraging each other to model the situation to get started.
- When some students heard what other students had ideas about, other students seemed more willing to share ideas.

Implications for teaching beyond this lesson

- Asking students to build on each other's ideas promotes the idea that mathematics is about reasoning and sense-making.
- Giving students the space to reason and share their thoughts allows them to identify constraints and important elements of a problem.
- Re-voicing and/or recording the words or ideas that students' use (high or low status) provides legitimacy to that student's ideas in front of his/her peers.

Hypothesis 2 – Shifting attention towards general relationships

Questions should shift students' attention away from the computation towards general relationships and properties.

Observations

- Modeling students' thinking on the table during the warm-up may have supported students in producing tables to work on the main task.
- In the warm-up, students were asked to talk about the difference between the 2 models of bacteria they had an opportunity to compare linear vs. exponential relationships.
- When 2 sketches of graphs were displayed, students were asked to turn and talk about "how is the rate of change or change over time different in these graphs?"
    - A is a constant rate of change, B is a swoop
    - This is not super constant.
    - In B it takes a little bit to get going, but then it starts going fast.
    - Haven't you ever seen what happens when things start to double.
    - It's going to be really steep.
    - Predicting what the graphs will look like "outside the viewing window"
- Students were focusing on relationships when asked to talk about how you would know the amount of money for any day number.

Implications for teaching beyond this lesson

- Taking off specific information to find (how much money would there be in __ weeks) might allow students to look for general relationships more than solving using computation.
- Removing the questions allowed students to think about and generate what you could figure out from the information given in the problem.
- Students can ask questions of each other and push each other to talk about why they are doing what they are doing, which can lead to discussions about general relationships (sketch of graph connected to an equation).
- Because students had time to investigate the task, they were able to look for relationships.

Hypothesis 3 – Listening to and critiquing each other's arguments

Listening to and critiquing arguments supports students in learning new argumentation techniques and considering ideas other than their own.

Observations

- Students having time to work in small groups allowed them to listen to each other's thinking and ask questions of each other.
- During the 3-Reads, students used the sentence stems which generated discussion about the different possibilities in solutions to the task.
- Allowing students to start on their own allowed for opportunities to share differences in their thinking.
- The students were able to ask each other why they were thinking what they were thinking because the task was more about thinking than computation and procedures.

Implications for teaching beyond this lesson

- Tasks that force a choice between a few possibilities may lend well to argumentation.
- Using routines that are open-ended allows for argumentation while also setting the stage for using that same routine for argumentation purposes. Knowing that there is not a "right answer" promotes students' willingness and desire to contribute to the whole group discussion.
- This regular practice may support students in valuing each others' thinking.
- Hearing what other students say is likely to support students' confidence when they hear what other students are claiming.
- It is likely to be useful to give students time to revise their thinking after they get feedback from other students.

Hypothesis 4 – Re-voicing and having students re-voice each other

By focusing the discussion around thinking (not just doing) and teacher's strategic use of language, students can come to view mathematics as a process of collaborative exploration and explanation.

Observations

- One student shared in the whole group discussion how he changed his thinking which seemed helpful in setting the tone.
- The teacher called students' models/strategies "ideas" which also seemed to set the stage for seeing mathematics as a process of collaborative exploration.
- One student asked the teacher "what are we supposed to do?" The teacher's response was to "Write down whatever you're thinking", which then produced writing out their thoughts.
- The focus of the discussion was on the students' thoughts, not the teachers' by saying things like "maybe you can organize your thinking in a table".
- On several occaisions, the teacher displayed students' strategies/models and asked others to interpret them.

Implications for teaching beyond this lesson

- Allowing a variety of students' ideas come out in discussion, students come to realize they can contribute to each others' thinking and learning.
- When we don't make the students think that we are expecting a particular answer, the students are more willing to work and share their ideas.
- Allowing students to make sense of the context and use sentence stems during the 3-Reads routine, kept the discussion on thinking and off answer-getting.

Hypothesis 5 – Changing communication contexts

Small group discussions (and, at times, independent thinking) prior to whole group discussions will support students to view themselves as thinkers and doers of math.

Observations

- By allowing students to talk in small groups, students were much more willing to ask questions of each other and try to interpret each other's thinking.
- Students seemed motivated during the small group discussion to engage with other students' reasoning.
- Using students' representations to be displayed in whole group discussions could also be movitating for students to continue problem-solving through the task.
- Students of low status seemed to have more opportunities to work on the task and make attempts at problem-solving because the small group discussions were lower risk.
- Low status students had less space to "hide" than in the whole group discussion.
- It seemed apparent that the whole group discussion created a higher risk situation for students to contribute.
- Warm-calling students during small group discussion allowed students to mentally prepare for sharing in the whole-group discussion.

Implications for teaching beyond this lesson

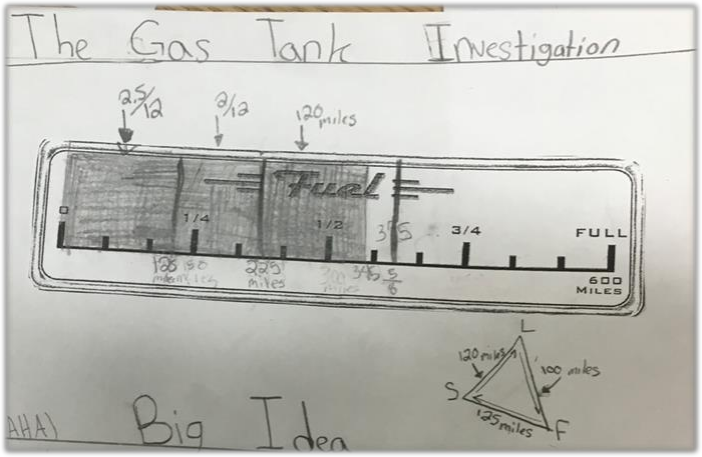
- Students are going to be more willing to share in the small group discussions than whole group discussions.
- Giving time to discuss in small groups will also increase the likelihood of low-status students to contribute in the whole group discussion.
- Pairing students so they push each other along is useful in executing this teaching practice.
- Having small group discussion (and/or asking assessing reasoning questions) prior to whole group discussion allows the teacher to "warm-call" students to position them as thinkers and do-ers of mathematics.
- Small group discussions will allow more variety of ideas to be shared.

Supporting Argumentation and Precision Lesson Study

January 29, 2020

Research Question:

Supporting students to develop and defend convincing arguments while attending to precision



The lesson study team hypothesized that the following actions would be important elements in supporting argumentation. Each hypothesis is listed below followed by the team's reflection.

Hypothesis 1 – Shifting attention away from computation

The teacher can use questions that shift the focus from computation with numbers to noticing and articulating the properties and relationships of these numbers.

- When the teacher asked for clarification on how students knew ¼ = 3/12
- During the investigation, students were discussing general relationships and using proportional reasoning to think through the problem.
- It seemed evident that many students had constructed generalizations about fractions that they were using in their choice of strategies.
- Students were using the number line and double number line to support their generalizing of the mathematics.

Hypothesis 2 – Collectively building arguments

Students should be collectively building arguments as a team as a way for them to understand that we can respect each other for the innate abilities that we have.

- The teacher re-voiced, had students re-voice each other, created opportunities to engage with each other's reasoning and changing from small group to whole group were important moves to collectively build arguments in the congress.

- Asking students to convince their partners during the investigation is one way to support students in engaging in collective argumentation.

Hypothesis 3 – Supporting the unpacking of ideas

The unpacking and development of ideas is important in supporting students' attempts at argumentation and proof.

- A culture of reasoning and argumentation will support the likelihood that students will unpack and develop their ideas.
- Having students ask each other questions in the congress supports the unpacking of ideas.
- Having access to model the situation with the gas gauge (which was close to a double number line) provided access to unpack mathematical ideas.
- Modeling thinking on the ratio table was also useful for articulating their mathematical ideas.

Hypothesis 4 – Using flawed claims

Teachers should use/leverage flawed claims to surface for discussion and intentionally focus on them.

- Modeling students' thinking during conversations might be helpful for students to see the flaw in the reasoning.
- Asking students during the investigation to refer back to the context to think more about a flawed idea was supportive of examining more of the relationships in the problem.
- Some students were thinking that each distance was 120 miles, which was used in the congress to discuss why it would not be true.

Lesson Study Models for Thinking about Percentages

February 6, 2020

Research Question:

How can teachers support students in developing the use of mathematical models as tools for thinking and deeply understand them so that they can be used to stimulate meaningful discourse?

The lesson study team hypothesized that the following actions would be important elements in supporting the development of models as tools for thinking. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Modeling students' thinking

When students see their teachers using models to represent their thinking over and over again and when they are allowed to investigate rich contexts where these models arise naturally, they transition from models of thinking to models for thinking.

- One student chose to use the constant of proportionality to think about the percentage and relationship.
- Probing thinking when students are sharing their ideas may allow more students to justify thinking and reasoning on the model we are using for students' thinking.
- The use of turn and talks while modeling students' thinking during the string was useful in making sense of the situation.
- The teacher was re-voicing students' ideas while modeling students' thinking.
- The sequence of expressions in the string was important in using the double number line as a tool for thinking.

- Modeling students' thinking allows the teacher to withhold the authoritative stamp of approval and allows the math to speak for itself.
- The teacher was able to model students' thinking on the double number line who had inaccurate reasoning, which allowed the student to see the fallacy in his own thinking (instead of the teacher pointing it out).
- The teacher can continue to press students in describing where their ideas would be in the model to support their reasoning while simultaneously promoting the use of the model.

Hypothesis 2 – Progression of models

The progression of models and modeling includes models of a situation, models of students' thinking, then models are used as tools for thinking.

- In this case, students did not experience the double number line from a situation, but it did not impede many from using it as tool for thinking.
- The students had been using double number lines prior to this week, but were having their thinking modeled on the double number line during strings for the 3 lessons prior.
- The rigor of the problem promoted the need to use the double number line to think with.

Hypothesis 3 – Shifting attention towards relationships

By asking questions to shift students attention towards relationships between their models, students' construct new mathematical objects and engage in iterative cycles of modeling (whole group and in pairs).

- Students were revising without prompting from the teacher.
- The teacher was drawing students' attention to the models they were constructing and asking them to interpret the reasoning within the model.

Hypothesis 4 – Withholding stamp of approval

We need to withhold authoritative stamp of approval and give the mathematics back to the kids to reflect on and sort out.

- During the problem string, the teacher modeled students' thinking, re-voiced students' ideas, probed student thinking and asked if what he modeled matched their thinking.
- The double number line seems to provide more evidence as to reasonableness than ratio table might because magnitude needs to be considered in constructing a double number line.

Hypothesis 5 – Congress after the game

Students need opportunities to communicate the meaning of their own and interpret other students' models.

- The gallery walk seems to support the idea that their audience is the other students.

- Many of students' comments were about right answers or procedures.
- Giving students an opportunity to revise their arguments after the gallery walk without erasing is something this group is interested in.
- The use of sentence frames should support less focus on on the computation in each other's reasoning/thinking.
- In this congress, we did not exactly connect the ideas to the game but used some of what they were trying to do to deepen their understanding of equivalence.
- It seems more useful to consider the big ideas that we want the students to discuss than having the actual questions that we "should" ask.
- In this congress, we presented hypothetical situations that could have or did occur in the game.
- We can sequence situations from the game in a way that we might do during a string of related problems.

Lesson Study – Questioning with Fractions

February 10, 2020

Research Question:

How can teachers facilitate and/or support meaningful, student-centered discourse while posing purposeful questions that advance all students' reasoning?

The lesson study team hypothesized that the following actions would be important elements in questioning that supports meaningful, student-centered discourse. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Celebrating, getting underneath and challenging

Questioning that advances students' reasoning involves the teacher considering what the children are trying to do, celebrating and getting underneath what they've done or are trying to do, then challenging.

- The students in this classroom have shifted away from answer-getting and see themselves as know-ers and do-ers of mathematics.
- The teacher can use wait-time to allow students to reflect on their own thinking.
- Asking students about how multiplication and division are related and what do the quantities in their thinking mean shifted students' attention away from computation towards general relationships.

- Asking students what they're noticing and make connections allowed students to build on each other's ideas.
- The teacher used the teacher discourse moves of wait time, probing thinking, re-voicing, having students re-voice were all critical in keeping the discussion student-centered.

Hypothesis 2 – Understanding and use of trajectories

Teachers' deep understanding and use of a reliable learning trajectory is important for posing purposeful questions to advance students' reasoning.

- Keeping the big ideas in mind will support teachers in becoming increasingly more powerful questioners.
- Collaborating across grade levels is crucial for understanding what big ideas, strategies and models to go after and why.
- It was clear that the teacher was thinking about the landscapes during this lesson which prompted stronger questions.
- Having contexts in sequence along trajectories makes the teaching and learning of the most important mathematics more efficient.

Hypothesis 3 – Considering who the audience is

Having students consider what mathematical idea they are trying to convince their audience of before going to the gallery walk/congress helps students to consider the perspective of their audience.

- Students are starting to realize that they are responsible for convincing each other.
- There are lots of opportunities for teachers to emphasize that the other students are their audience.
- Mathematical claims are the big ideas or generalizations that we would like students to discover, prove, and bring to the congress.
- The teacher can elevate students' status in front of their peers by warm-calling students from the discussion in the turn-and-talks.
- The students in this class seemed to know that their audience is the other students, not just the teacher.

Hypothesis 4 – Re-examine, revise and simplify ideas

Students need multiple experiences to re-examine, revise and simplify their ideas (before/after gallery walk) to build a foundation for understanding a mathematician's view of proof.

- Having students work on a rough draft first and then revise their work to make their poster about one or two big ideas.
- When we got to the congress, students were asked to reflect on the investigation and string of related problems to connect to the congress discussion.

- The congress supported the opportunity for students to re-examine, revise and simplify their ideas and the ideas of other students.
- The gallery walk and congress both allow students to bring ideas to the community to vet.
- The students were given an opportunity to switch math partners during the congress to get up and share with a different partner.

Hypothesis 5 – Modeling students' thinking

Teachers can model students' thinking and the processes they use so they have objects to discuss and can examine their logic.

- The modeling of students' thinking was critical for students to examine their thinking and logic.
- As the teacher modeled students' thinking, she kept saying "are you saying…" which was getting underneath their thinking.
- The way that we model their thinking will influence the models they choose to use over time.
- Students will inherit the models that we model their thinking with and use them as tools to think with over time.

Lesson Study Games, Number Sense and Fluency

February 12, 2020

Research Question:

How can we simultaneously develop students' number sense, procedural fluency and automatization of the basic facts?"

The lesson study team hypothesized that the following actions would be important elements in supporting number sense, procedural fluency and automatization of the basic facts. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Modeling students' thinking

Teachers can use re-voicing (modeling student thinking) and questions to support students in refining their mathematical language to develop language appropriate for argumentation within their community.

- The teacher was modeling students' thinking throughout the game (number rack and ten-frame) and the congress (equivalent expressions and rack).
- It seemed critical that the teacher was modeling the students' thinking.
- Modeling of students' thinking allows for the teacher to continue ask probing thinking questions to dig deeper into the mathematics.
- Having the equivalent expressions on the board may have left residue for the students to see relationships between the numbers and expressions and discuss their equivalence.

- Having a congress after the game allowed for modeling students' thinking to strengthen the power of use of games.

Hypothesis 2 – Teacher discourse moves

Teachers can use re-voicing, asking students to re-voice (during game/congress) and wait time 1 and 2 (during congress) to structure and focus the conversation on an important idea.

- Changing the communication context between small group and whole group allowed students to discuss their ideas, and the ideas of their students.
- The teacher regularly re-voiced students' ideas and asked students to connect to each other's thinking.
- By asking questions that drew students' attention towards treating expressions as object seemed to shift their thinking from counting on during the game towards using expressions as objects during the congress.
- Re-voicing students ideas while modeling their thinking seemed to draw attention towards equivalent expressions (more than counting on).

Hypothesis 3 – Developing a habit of always questioning students

Teachers can develop a habit of always questioning students, whether correct or incorrect, to help students learn to dig deeper into an idea and unpack concepts and develop the skills related to proving.

- Probing students' thinking brought out a variety of students' thinking during the congress.
- Asking students questions about their ideas when they were already correct allowed for deeper discussion of equivalent expressions because a variety of ways of thinking were made visible to the whole class.

Hypothesis 4 – Habit of always questioning

During the cooperative game (not competitive), teachers should observe strategies students are using and confer during the games just as we would during an investigation - looking for landmarks emerging on the landscape, conferring on strategies they are trying to do.

- The context of the game allowed for students to use strategies they wanted to use and the teacher to observe and authentically assess where they were working on the landscape.
- Students were not competing with each other in this game, but were trying their own strategies and at times working cooperatively but other times working independently.
- There were instances during the game when students were working cooperatively to support each other's thinking.

Hypothesis 5 – Congress after the game

We can hold a congress after kids play games focused on strategies they tried, strategies they found helpful or insights they had while playing the game while focusing on pushing from their ideas to what's on the horizon.

- In this congress, we did not exactly connect the ideas to the game but used some of what they were trying to do to deepen their understanding of equivalence.
- It seems more useful to consider the big ideas that we want the students to discuss than having the actual questions that we "should" ask.
- In this congress, we presented hypothetical situations that could have or did occur in the game.
- We can sequence situations from the game in a way that we might do during a string of related problems.

Hypothesis 6 – Using landscapes/trajectories to focus on development

Teachers can interpret student thinking in terms of reliable trajectories in order to shift away from a deficit mindset way of thinking.

- Recognizing/identifying where students are in development allowed our thinking to be around what they are doing instead of not doing.
- Having trajectories that identify important milestones makes it more reasonable for the teacher to know what might be next in a students' development.
- Understanding the most important and foundational mathematics will serve students well in later grades.

Building a Mathematical Community Lesson Study

February 26, 2020

Research Question:

How can teachers support students to see that being in a mathematical community involves contributing to conversations about disciplinary ideas, to build on others' ideas and have others build on theirs?

The lesson study team hypothesized that the following actions would be important elements in building a mathematical community to enact over a period of months. Each hypothesis is listed below followed by the team's reflection.

Hypothesis 1 – Routines

The feedback enabled by routines will help students revise and refine not only the way they organize and communicate their own ideas, but also ask questions to clarify their understandings of others' ideas.

- In the homework discussion, some students were revising their answers.
- Some students were checking each other's answers saying things like "what did you get?"
- Giving students time to think on their own and talk in small groups before talking in the whole group seems to allow students the opportunity to revise their own and others' thinking.

- Some of the routines seemed to provide an opportunity to communicate with each other to bring their thinking closer together.
- Notice/wonder and independent think time with small group discussion, allowed a greater level of safety for students to explore ideas.

Hypothesis 2 – Temporary supports

Temporary supports, or scaffolds, can include teacher modeling, supporting students in making charts with mathematical information from a word problem, providing manipulatives or graphic organizers to support sense-making, identifying and drawing upon students' inner resources, and structured peer interactions.

- Allowing access to Chromebooks in 1 per pair seemed to have potential to support sense-making.
- The teacher modeled students' previous thinking from a lesson earlier in the unit to have greater access to ideas in today's problem.
- It might be important for teachers to consider which ideas should be preserved on longer-lasting artifacts (chart vs whiteboard).
- Teachers can use the document camera to share representations of what students have produced to shift the authority back to the students.

Hypothesis 3 – Implicit messages

Implicit messages must be dealt with explicitly if we hope to improve the positioning of all students relative to mathematics (pointing out what mathematicians do, homework, grades, assessments)

- Despite the homework discussion having potential to be procedural and focused on the teacher, the teacher was able to use routines and discourse moves to focus on the reasoning occurring in one of the homework questions.
- Phrasing questions and what gets re-voice places value on reasoning and problem-solving, revising their thinking.
- What the teacher does with the task is important to determine whether students value reasoning or answer-getting.
- The teacher was re-voicing students' ideas in an attempt to amplify the mathematics and language.
- The teacher repeatedly collected students' ideas that were being developed in the small group discussions that were consistently used in the whole group discussion.

Hypothesis 4 – Doing and thinking tasks

Requiring both of these types of processes (doing and thinking) has the potential to position students as doers and thinkers while simultaneously positioning mathematics as a discipline in which actions are coupled with reasons and justifications.

- Students seemed to naturally follow their statements with reasons.
- Providing individual think time before discussing supports recognizing mathematics as a thinking discipline.
- During the notice/wonder, one student expressed concern that she was making an observation without a justification which could be evidence that students are experiencing both doing and thinking tasks.

Hypothesis 5 – Connecting conceptual understanding with procedural fluency

Connect conceptual understanding with procedural fluency to reduce mathematical anxiety and position students as mathematical knowers and doers.

- During the discussion about the homework question, one student used understanding of what logarithms are to use logic for why the reasoning of the hypothetical student could not be accurate.
- The homework question that exposed fictitous reasoning based on a common misconception provided an opportunity to connect conceptual understanding to procedural fluency. This also allowed students to recognize the need to generate new mathematical language/notation.

Hypothesis 6 – Elevating socio-mathematical status

Make student thinking public, and then choose to elevate a student to a more prominent position in the discussion by identifying his or her idea as worth exploring, to cultivate a positive mathematical identity.

- The small group discussions allowed the teacher to "warm-call" students and ask them to share their ideas in the whole group discussion.
- During the "Discussion Supports" routines using the teacher discourse moves of re-voicing there were a few students that began to add on to each other's ideas in the whole group discussion.
- Students of low socio-mathematical status were given an opportunity to start the discussion on important ideas, but it seems critical for the teacher to be patient to allow these students the time to develop their ideas in their small group discussions.
- The teacher was prioritizing involving students of low status in the whole group conversation, which kept those students involved in contributing ideas.

Precise Measure of Rotation Lesson Study

February 28, 2020

Research Question:

Supporting students to develop and defend convincing arguments

The lesson study team hypothesized that the following actions would be important elements in supporting students to develop and defend convincing arguments. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Modeling students' thinking

We need to model students' thinking and the processes they use so they have objects to discuss and can examine their logic.

- The modeling of student thinking in many, many cases were done to illustrate the rotation over a pivot point with either a marker, body position, vision line and skateboard.
- The students were regularly communicating with each other using the same type of modeling about rotation and angle measure.
- Modeling students' thinking may help to support moving from horizontally mathematizing (from the world of life and the world of symbols) and vertical mathematizing (within the world of symbols).

Hypothesis 2 – Celebrating, getting underneath and challenging

Conferring with students by intently listening to what they are doing or trying to do, celebrating, getting underneath their strategy and challenging will be important for preparing students for a meaningful whole group discussion.

- The teacher consistently found ways to celebrate their thinking in each conferral which made it more likely to persevere.
- This structure may also support the development of relationships with students.
- The students recognize that they own the mathematics and that we are all learning the mathematics more deeply during the investigations.
- The teacher seemed to be able to find an entry point in the discussion and was able to nudge their thinking.
- Keeping students grounded in the context was a helpful part of the conferrals.
- Keeping a few big ideas in mind during the conferrals might help the teacher confer more productively.

Hypothesis 3 – Re-voicing, asking students to re-voice each other

Re-voicing students' ideas and/or asking students to rephrase what one of their peers just said in their own words allows students to hear some of the important statements several times to understand their significance and can lead to asking students whether the other students agree or disagree.

- During the congress students were given opportunities to turn and talk about what other students said.
- During the reflection, the students had opportunities to re-phrase how they interpreted the summary of the congress.
- Re-voicing and having students re-voice in the conferrals seemed helpful in keeping both students in the partnership involved in the conversation.
- We can ask students what they think about what their partner thinks after re-voicing.
- Having students re-voice each other in the congress might be useful in keeping all students in the conversation.

Hypothesis 4 – Noticing and articulating mathematical ideas

The teacher's role is to help students notice and articulate mathematical ideas, support the development of mathematical language and help students unpack their ideas whether incomplete or even inclusive of faulty solutions.

- Most of the students were more focused on the math than on finding the answer to today's problem.
- By regularly supporting students to focus on and notice mathematical ideas, it's an intentional move away from "answer-getting" and promotes more sense-making.

- Modeling students' thinking plays a role in supporting the development of mathematical language (rotation, angle, vision line, protractor).
- The students did not seem concerned that they did not know how to use the protractor indicating a productive disposition towards doing mathematics.

Hypothesis 5 – Moving the dialogue ball between students

Moving the dialogue ball amongst the kids because it is the group's responsibility to vet other student's ideas, examine them and decide whether to accept them into the community.

- The teacher re-voiced a lot of students' thinking during the congress but it wasn't clear who was owning the ideas that were being re-voiced.
- During the conferrals, it would be good for the teacher to support students in knowing what they should share in the congress.
- Asking students to produce a "claim" on their poster can help keep the congress focused on the mathematical ideas and give the presenters a starting point.
- There is some potential for having "pre-congresses" where partnerships of students get together and rehearse for the congress and get feedback and questions from the other group to give more students time to think about their ideas and how they can communicate them.
- A variation on the gallery walk would be to move the posters between the groups, give them individual think time to read the poster, discuss with their partner (possibly using sentence frames), collaborate about comments they can make, etc which seemed to be a useful model to structure their feedback.
- Having students re-voice each other's ideas prior to documenting the generalization from a poster might help to keep the vetting of the ideas on the students.

<u>Individual team-member take-aways</u>

- Having students reflect on their ideas in a journal at the end of the day can help to make their learning a little more concrete.
- Holding a pre-congress with students would likely contribute to more powerful congresses.
- Documenting students' claims to summarize the congress
- Variation of gallery walk to move the posters around the room instead of the kids moving around the room
- Using math journal or reflection prompts at the end of the class period
- **The group is very interested in considering how we can use reflection throughout each unit.**

Tasks that Promote Reasoning and Problem-Solving

March 1, 2020

Research Question:

How can teachers implement tasks that promote reasoning and problem-solving to advance students' reasoning and develop number sense?

The lesson study team hypothesized that the following actions would be important elements in implementing tasks that promote reasoning, problem-solving and number sense. Each hypothesis is listed below followed by the team's reflection.

Hypothesis 1 – Treating mathematics as a social activity

Education should offer students opportunities to share their strategies and inventions with each other. By listening to what others find out and discussing these findings, the students can get ideas for improving their strategies. Moreover, the interaction can evoke reflection, which is necessary to reach a higher level of understanding.

- Because the task required that they worked in pairs forced them to reason and problem-solve together. Treating doing math as a social activity.
- Given that there were numerous possibilities for strategies allowed for more reasoning during the investigation in their pairs.
- The strategies in the congress were higher in development than occurred during the investigation.
- The congress is critical in advancing students' reasoning.

- Modeling of students' thinking during the congress allowed more students access to the mathematical ideas others were sharing.
- Students were trying to use relationships and what they know about addition to place numbers on the number line.

Hypothesis 2 – Starting with a context with multiple entry points

Starting point is the exploration of a context problem which — and this is very essential — can be solved on several levels of understanding.

- During the conferrals the teacher consistently kept students grounded in the context.
- All students were able to start working on the problem without any support from the teacher.
- Having a discussion before students began the investigation to develop the context was helpful in giving students access to getting started on the task.
- Teachers can take students' ideas, re-voice them and refine language that might be useful in working on the forthcoming investigation.
- Immersing students in the context of a story allowed students to have a common starting foundation to think from.

Hypothesis 3 – Students actively develop tools and insights

The students, instead of being the receivers of ready-made mathematics, are considered active participants in the teaching-learning process, in which they develop mathematical tools and insights.

- The teacher re-voiced students' ideas in the conferrals and during the congress, really trying hard to build on their ideas and make it known that their ideas are valued.
- Allowing the students time to get started on the task while the teacher is conferring with other groups gives students the space to struggle productively.
- Starting the congress with a students' way of thinking that most students can make sense of will support engagement in the discussion.
- The task was open enough for students to be allowed to use strategies that make sense to them, did not elude to (or require) a preferred strategy or model, which allowed them to own their ideas.
- The congress should allow students to bring their ideas to the group to be considered, thought about, modeled and allow other to see a different perspective or think about something they had not thought about.
- Teachers can take advantage of opportunities to push students' ideas back to the rest of the group to think about.
- Having students turn and talk allows the teacher to elevate students with low-status by asking them to repeat what they said in the whole group.

Hypothesis 4 – Advancing reasoning by generalizing

Advancing could include examining structure and regularity, finding ways to make the strategy more efficient, examining the problem with different models as tools for thinking, generalizing beyond the problem or writing an argument to convince others.

- "See if you can use her thinking to build the next one" was a move to extend from another students' thinking and make the strategy more efficient.
- The teacher continued to walk away after asking questions that advance students' reasoning to promote deeper thinking or making strategies more efficient.
- Getting underneath's challenges in inefficient strategies might help them to be more willing to take on the advancing reasoning questions/ideas.
- The ideas that students were coming up with during the congress were moving most of the class along in trying to use more strategies.
- Asking advancing reasoning questions during the conferrals is only part of advancing reasoning, but the congress and other routines could support as well.

3rd Grade Fractions Lesson Study

March 4, 2020

Research Question:

How can teachers use tasks that allow all students to reason and problem-solve and pose purposeful questions to advance students' reasoning towards more efficient strategies?

The lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to advance students' reasoning. Each hypothesis is listed below, followed by the team's reflection.

Hypothesis 1 – Conferrals to celebrate, clarify then advance reasoning

Conversations should start with celebrating their strategies or what students are trying to do, then re-stating or clarifying ("are you saying") and advancing their reasoning.

- The teacher was clarifying/re-voicing and celebrating what students were trying to do during the conferrals.
- During one conferral, students started on their own with their idea. The teacher celebrated their idea, but also helped them to see the fallacy in their reasoning, which prompted students to try another strategy.

- Use of the landscapes can support teachers in celebrating, knowing what to clarify and where to challenge.
- The teacher treated the conferrals as conversations rather than monologues.

Hypothesis 2 – Convincing others and generalizing

Students should be asked to try to convince others that their reasoning is sound, or that their strategy will work in other situations.

- During the investigation, students knew they were supposed to convince each other of their reasoning.
- Students were trying to convince each other of their ideas.
- In the congress the students were asked to communicate with each other (not just the teacher).
- During the gallery walk, the posters were called arguments and students were given time to write comments and get feedback on their posters.

Hypothesis 3 – Using teacher discourse moves

Questions and teacher discourse moves should be used to support students in discussing their ideas with each other.

- Re-voicing and having students re-voice occurred numerous times during the number string, which set up the opportunity for students to add on to each other's thinking.
- There were numerous opportunities that the teacher used to have students turn and talk about what other students said.
- Having students re-voice each other showed students they were responsible for listening to each other.
- Warm-calling students allows students to prepare to share in the whole group discussion, and at times, some of the specifics of their ideas to be shared.

Hypothesis 4 – Shifting attention towards relationships

We need to shift students' attention away from computation/procedures towards noticing and articulating properties and relationships.

- The string of related problems allowed for multiple opportuntities to discuss general ideas and relationships (commutative property, inverse relationship between multiplication and division, equivalence)
    - Could you write the inches as a fraction?
    - Why is 4 inches the same as 1/3 of a foot?

Hypothesis 5 – Vertical mathematization

It is important for students to operate on their own reasoning to generalize or mathematize vertically.

- “What does the total represent? What is this equal to? What do you mean by that? How do you know…?
- Mathematical models bridge between the context and generalizing the most important big ideas.