

8th Grade Lesson Study – Linear Relationships – Stacking Styrofoam Cups

September 19, 2017

Research Question:

How can teachers use and connect mathematical representations to facilitate meaningful discourse in supporting productive struggle for all students?



This lesson study team consisted of three 8th grade teachers, two 9th grade teachers, and a district instructional coach. All team members are from the same district, with one member being on a different junior high school campus. The shared understandings are broken into two components – important observations and implications for teaching.

Important observations:

- Students had difficulty connecting some of the representations that were displayed initially. The more time students had to analyze the representations, the more they seemed to be able to make sense of them.
- Students were able to connect the visual representation to the table, but had more difficulty connecting the table to the equation. One student mentioned she “didn’t know where to put her eyes” when a representation was displayed.
- No students produced a table to represent their way of thinking.
- Many students wrote a verbal description of the process they used to come to an answer.
- Students produced more rich thoughts in small group or individually than they did in whole group discussion.
- The reason (perceived ability, social dynamics, etc.) for putting students together within small groups played a role in the meaningfulness of the student discourse.

- Students planned a solution pathway rather than jumping into a solution attempt.
- Many students were paying particular attention to the precise measurement of the “brim” of the cup.
- Many students had intuitive ideas of the rate of change in terms of this context.
- There were many times where small groups talked for a minute or two, but then were quiet or off-task until the next question.

Implications for teaching:

- As teachers select and sequence student work, they must consider whether to display a student’s entire representation or only certain features of that representation to connect the mathematical ideas between and within each. Teachers can reveal more details subsequent to class discussions or ask the student author if necessary, but too much work on a page may distract from the mathematical features intended for highlighting.
- It may be necessary to ask students to produce a different representation than they have already used. Students’ representations for a solution path or an explanation may differ from representations they use to actually solve a problem. The question that teachers use to ask for this work may influence the representation that is produced.
- The teacher’s role during small group discussion time should be more in terms of monitoring than questioning individual students **if** the teacher is planning on having a whole group discussion of the ideas coming out in the small group discussions. The teacher should provide students time and rehearsal for moving private ideas to small groups and then to whole groups because individual student work may be richer than is shared with the whole class.
- Students may need explicit teaching and practice including structures to support how to listen to each other’s ideas in order to develop meaningful discourse that supports productive struggle. Without support, student sharing may wander into a variety of topics or be a telling of what each student did without any understanding of each other’s thinking. One possible accountability strategy to support students in understanding each other’s thinking is to have students interview each other and take notes about what each other’s is thinking.
- Writing down what students notice in the beginning of the lesson may be helpful in using those details and referring back to them once students begin working toward a solution. Making the noticings public and preserving them on the whiteboard keeps them available for all students to use throughout the lesson.

Questions for further study:

- What are important features of tasks that would lend themselves more to students producing tables, graphs or equations?

6th Grade Lesson Study – The Folded Number Line

September 27, 2017

Research Question:

How can teachers use small group discussions to build shared understandings in facilitating whole group discourse?



This lesson study team was most interested in supporting students in leveraging the powerful discussions they have in small group discussion (with about 4 students per group) to support whole group discourse. Prior to this lesson study cycle, all teachers on this team were finding that, while students were willing to discuss mathematical ideas in their small groups, students' desire to share those ideas out in the whole group setting was low.

Upon researching this topic, the team began hypothesizing that the use of these teaching practices or strategies will support students in using their small group discussions to build shared understandings in whole group discussions:

- Beginning the task by asking students what they notice and wonder about an image slightly connected to the mathematical topic, yet somewhat vague and ambiguous
- Using talk moves, or teacher discourse moves, (Chapin, O'Connor, Herbel-Eisenmann), throughout the task, including:
 - Waiting for students to think about a question posed by the teacher, but also waiting to think about another student's response before responding
 - Revoicing – teacher revoices, re-states or rephrases a student's thinking
 - Asking students to revoice another student's thinking in their own words
- Use of visual and physical models
- Anticipating, monitoring, selecting, sequencing and connecting of students' representations (5 Practices for Orchestrating Productive Discussions, Smith and Stein)
- Using a focusing pattern of questions while avoiding a “funneling” pattern of questions

- Explicitly discussing the value of whole group discussions with students

Important observations:

- When asked in the beginning of the lesson if other groups should hear the ideas they come up with in their small group discussions, many students said that would be “cheating” or inappropriate in some way. However, later in the lesson, many students articulated the opposite, that sharing ideas from their small groups with the whole group was helpful for everybody.
- Establishing some of the explicit language to structure the small group and whole group was helpful. Making students aware that they were going to share their ideas from their small groups in the whole group setting seemed to support students’ willingness to share ideas with the whole group.
- The teacher was pleasantly surprised as to how much students engaged the writing portion, and stated the quantity and quality of writing was significantly more than usual.
- After students had a chance to talk in the small groups, at least one student in every group had a hand up almost every time, which was different than most other days.
- The more extensive the small group discussion was, and time allocated to do so, the more students seemed willing to share those ideas with the whole group.
- The students seemed willing to share not just what they thought, but their reasoning behind what they thought.
- Having the models and tools (visual representations) were extremely important in students’ contributions to both small group and whole group conversations.
- The students that are typically reluctant to share were willing to share their thinking with their group.
- The observing teachers noticed several students self-correct thinking, or revise thinking as a result of small group conversations.
- The group noticed that when we asked questions that had more of a one-word answer (funneling pattern), there were many less students willing to share their thinking.

Implications for teaching and learning:

Asking students to notice and wonder can be used to provide students with an opportunity exercise think-pair-share in a safe environment, and engage in a discussion that allowed them to use creative thought, even when the link between the actual object to notice and wonder about and the lesson is somewhat obscure.

Re-voicing to clarify what students meant seemed to validate some of the students’ thinking and is a teacher discourse move that all members of the team felt would consistently support moving from the small group discussions to whole group.

Asking for students' *thinking*, instead of what their answers are, is more likely to allow students to contribute to whole group discussion. Closed questions, or questions that had limited responses seemed to restrict students' willingness to share their reasoning with the whole group.

Allowing students time to think, write their thoughts down, and discuss in small groups before discussing in whole group are absolutely critical to the quality of whole group discussions.

The use of visual representations clearly support whole group discussion, and should be used regularly by students.

While incredibly difficult, using a focusing pattern of questions certainly allowed for higher quality conversation for students in both small and whole group. When questions were in a funneling pattern, the conversation was less powerful. Additionally, when all students were asked to talk about ideas, rather than a conversation between the teacher and one student presenting his or her ideas, the reasoning of the whole class was advancing. Teachers should be aware that a one-on-one conversation with a student during whole group discussion time can lead to students disengaging from the discussion.

Having explicit discussions with the students about the power of their ideas for the benefit of the whole group should be made visible to students regularly to allow them to see and reflect on how their ideas are helping other students' mathematical understanding.

Ideas for further study...

How can teachers make the connections that students were making in small group discussions more visible to the whole group?

When is direct instruction (I-We-You) appropriate?

6th Grade Lesson Study – Tape Diagrams and Equations

September 30, 2017

Research Question:

How can teachers use purposeful questions to support the development of number sense (thinking flexibly about numbers and their relationships) while addressing middle school level content?



Upon researching through NCTM's Principals to Actions, the Expressions and Equations Progression Document, and the Illustrative Mathematics curricular materials, the lesson study team hypothesized that these actions would support posing of purposeful questions and the development of number sense while addressing grade level content:

- Providing students with multiple opportunities to share their thinking in small and whole group
- Providing opportunities for students to revise their thinking
- Allocating time for students to connect visual representations with symbolic representations
- Extending previous understandings of arithmetic, including the inverse relationships between addition/subtraction and multiplication/division, to algebraic equations
- Allowing students to go move both ways between visual and symbolic representations

Important observations:

When focusing patterns of questions were being asked, and students were given time in their small groups to discuss their thinking about those, some students revised their understanding or thinking of what the diagrams were representing.

When there was no small group discussion before whole group discussion, the level of thinking was minimal and students appeared disengaged.

When students were given opportunities to discuss in small groups, they had greater opportunities to think flexibly about numbers and their relationships.

When students were asked to describe what they noticed in the tape diagrams, students were able to make some connections between quantities, equations and the visual representation of them.

Students had significant difficulty in connecting the visual representations to the equations, illuminating potentially shallow understanding of the operations.

Students were making equivalent expressions showing the inverse relationships with multiplication and division but not matching them to the visual representations.

Early in the lesson, 14 out of 19 students were able to create tape diagrams when given $4+3=7$ and $4 \times 3=12$ when the teacher asked students to produce them, without telling them how to do it.

Implications for teaching and learning...

This lesson study team believes, as a result of this cycle, that the actions below will lead to teachers' ability to pose purposeful questions to develop students' number sense while addressing middle school level content:

- Providing students multiple opportunities to share their thinking in small and whole group
- Providing opportunities for students to revise their thinking
- Connecting visual representations with symbolic representations
- Allowing students to move both ways between visual and symbolic representations
- Intentionally bringing out students' misconceptions whether or not students share them with the whole group (discovered in the lesson, but not hypothesized in the co-planning phase)
- Planning tasks that allow students to connect prior understanding of arithmetic with whole numbers to expressions/equations with variables and rational numbers

Also, it was clear to this lesson study team that rushing through content is detrimental to understanding and developing number sense. Sufficient time needs to be allocated at all grade spans for the above teacher actions to effectively promote number sense while addressing grade level content.

9th Grade Lesson Study – Rearranging Formulas

October 10, 2017

Research Question:

How can teachers use and connect mathematical representations to support students in making sense of literal equations?



In this lesson study cycle, through reading of Principals to Actions' section on Using and Connecting Mathematical Representations and the Algebra Progressions Document, and through exploring a variety of existing tasks and lessons to address the Algebra standard regarding “rearranging formulas to highlight a quantity of interest”, the team formed several hypotheses to suggest certain practices would lead to making sense of rearranging formulas. Below, each hypothesized action is listed, followed by observations from the lesson and implications for teaching around that idea, as well as evidence observed that students were engaged in Standard for Mathematical Practice 1 (SMP 1) – “Students make sense of problems and persevere in solving them.”

Numberless Context

Beginning with a brief context that contains no numbers, nor implies any specific question or solution strategy (numberless word problem)

Observations/Implications: This context forced the students to talk about the situation without any pretense, helped students to become more familiar with the context, and helped students to identify the unknowns, without rushing into trying to “solve”.

Connection to SMP 1 – Students “started by explaining to themselves the meaning of the problem” and “analyzed givens, constraints and relationships”.

Students Develop Questions

Asking students to develop questions to answer through their wonderings about the context (rather than giving them a question at the onset of the launch of the task)

Observations/Implications: Allowing students to have discussions about what they wondered gave them a better idea of the context of the situation. The team noticed that this pedagogical move helped students to clarify the context, to realize they were thinking many of the same ideas at the start of the lesson, supporting perseverance, and to recognize ambiguity in the beginning of the lesson is appropriate and normal (entry point for all students).

Connection to SMP 1 – Students “started by explaining to themselves the meaning of the problem” and “analyzed givens, constraints and relationships”.

Solidify Ideas Before Sharing

In some cases, asking students to write their ideas down before sharing them with others

Observations/Implications: Writing before pair-sharing seemed to allow more voices to be heard, force more ownership of students’ idea, and helps to slow the pace so that students do not miss key ideas that are developing as the lesson develops quickly.

Connection to SMP 1 – Allowing students to write their ideas allows students to “make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt”, as well as give students opportunities to “ask themselves ‘Does this make sense?’”

Sharing in Small Groups

Embedding multiple opportunities for students to discuss their ideas in small groups prior to discussing with the whole group

Observations/Implications: The team noticed that this strategy promotes risk-taking amongst the students (easier to be “wrong” in a small group). It was evident that students were more apt to share in the small group, and even if they did not want to speak in the whole group, they had a chance to say what they were thinking and share their reasoning orally. The team felt strongly that this also allows for formulation of ideas to develop at the students’ pace and thinking of math as a ‘rough draft’ where revisions will be necessary. An added benefit of this strategy also showed that this allows the teacher to converse with small groups of students to understand more of what they are thinking.

Connection to SMP 1 – Discussing ideas in small groups allowed students to “monitor and evaluate their progress and change course if necessary.”

Analyzing Context Without Calculations

Pressing students to think and talk about relationships within the context (or problem), without doing any calculations

Observations/Implications: The team noticed power in the students' discussions about relationships within the context. At the same time, it was quite apparent that there was a significant difference between what students said (orally) and what they wrote. This implies the importance of paying attention to the methods of communication students are asked to use, and the strengths and weaknesses of each method (talking in small groups, talking in whole group, writing, etc). Also noticed was that when the teacher reiterated what students said about connecting the equations with the context, small group discussion took on, and almost mimicked that focus (connecting the equations to the context) in their conversations.

Connection to SMP 1 – This clearly allowed students to “analyze givens, constraints, relationships and goals.”

Discuss Connections of Representations

Devoting time to allow students opportunities to make connections between contextual and symbolic representations, to articulate those connections they see with others, and listen to the ways other students describe those connections.

Observations/Implications: Perhaps the most important pedagogical move in this lesson, was to focus on the connections between these representations, and dedicating sufficient time to do so. The team emphatically believes that making connections across representations is critical in deepening students' understanding of mathematical topics.

Connection to SMP 1 – Structuring the lesson around focusing students attention to connecting between the contextual, verbal and symbolic representations of this situation allowed them to “explain correspondences” between these representations.

Discuss Differences of Representations

Having students discuss differences amongst the symbolic representations (equations) after discussing connections between the symbolic and contextual representations.

Observations/Implications: It seemed that students needed more time of connecting the symbolic representations to the contextual representation to develop meaning of the rearrangement to be more proficient with rearranging formulas without a context.

Connection to SMP 1 – Because students had significant difficulty in this lesson connecting between equations, the need to devote time as stated in hypothesis #6 above was noted by the team as critical in students' ability to “identify correspondences between different approaches.”

Implications for teaching and learning:

In summary, this lesson study team felt that all of the pedagogical moves that were tested in this lesson are incredibly important in supporting students in making sense of literal equations, and likely many other challenging mathematical topics.

Pre-Calculus Lesson Study – Connecting Representations of Optimal Solutions

October 17, 2017

Research Question:

How can teachers use and connect mathematical representations that support students' conceptual understanding?



This lesson study team investigated various strategies to support students in making connections between the variety of representations to develop and advance reasoning that multiple students produced after being launched a task in which they were not told how to solve. Several observations were made, and implications for teaching derived from this cycle.

Important observations:

- The team felt the students mostly did what was anticipated in the co-planning phase of the cycle.
- Students seemed very willing to write down what they noticed in the visual.
- Students struggled with making the verbal generalizations from the visual representations making it difficult for the teacher to decide how much time to spend in this area.
- As a result of pushing students to make connections between representations, the team felt students will have a better sense of what's happening mathematically in the problem that they did not have prior to the discussions around connecting the representations.

- The visual representations that were selected allowed students to see some of the relationships between the quantities and the constraints.
- The table allowed for students to discuss characteristics of the quadratic function with the numbers in the table showing the symmetry of the graph.
- Students were referencing what the graph of the relationship would look like even though they were not asked to.
- The sequencing of students' representations were as follows:
 - 1st representation - allowed students to make sense of the parameters of the problem through a visual representation.
 - 2nd representation – another visual that showed what was happening in the context without being overwhelmed with quantities
 - 3rd representation – verbal generalizations that described the relationships between the elements in the visual
 - 4th representation - student's numeric equation that included the relationships from the visuals but in a different form (symbolic)
 - 5th representation – co-developed table using some of the student's representations or thinking to bridge the visual with a symbolic representation
 - 6th representation – verbal description of the relationship between length and area proposed by a student (as length or width decreases, area decreases)
 - Students were then asked to use what's been discussed to produce their own symbolic representations
- To connect representations, students were repeatedly asked to discuss similarities among representations and at times, to alternate the direction of the connections made among representations.

Implications for teaching and learning:

Student discourse simultaneously supports the advancement of student's thinking and productive struggle, and many opportunities for students to discuss their thinking in small groups and whole groups is critical.

Starting with a context, then connecting the context to a visual representation of the context gave students a way to generalize the relationships between the variables. This sequence could be replicated for many mathematical topics.

Perhaps, the most impactful part of the lesson seemed to be students' interpretation of the Desmos visual that did not include quantities and allowed students to focus on the relationships between the variables. Introducing tasks or representations early in a task would support a focus students' thinking and reasoning on relationships more than procedures.

Splitting the lesson over 2 days where the first day students produced their initial representations, and the second day was devoted to discussion around using and connecting those representations allowed the team time to select and sequence the representations. The members of this team were interested in using that format in other lessons in their classrooms.

Those students that had internalized the connections between representations could more easily or naturally move between them, so allowing the time for them to internalize the connections, rather than simply giving them to students, is a critical aspect of understanding mathematical concepts.

Asking students how different representations are connected to each other is key, as it seems like there are various strategies that are **not** connected if students are not asked to describe the connections. When teachers do not facilitate discussion around making connections between representations, the lesson becomes more about “what’s the right answer for this problem” than generalizations that will help them solve other problems.

Listening to what students say in their small group conversations is critical for allowing the connections we want students to see between representations to be brought out into the whole group discussion. Specifically, when we want students to make generalizations, we want representations that allow for those generalizations.

Geometry Lesson Study – Slopes of Perpendicular Lines

October 20, 2017

Research Question:

How can teachers effectively use and connect representations (visual, symbolic and/or contextual) to deepen students' understanding of slope of perpendicular lines?



This lesson study team hypothesized the following teaching actions would support using and connecting representations that can be generalized to lessons that are both within and aside from the content being investigated in this lesson. Following are the hypothesized actions, observations the team made regarding each action, and implications for teaching based on those observations.

Visual Representations

Starting the task with dynamic visual representation, containing no symbols, and asking students what they notice and wonder...

Observations

- Students developed important questions.
- Allowed students to enter into the task
- Students felt there was a certain answer when asked what they wonder about the image.
- Students began to develop false generalizations.
- Students had 2 minutes to write down what they noticed, most of the students recorded deeper noticings after about 90 seconds.
- Students had 2 minutes to write down what they wondered.
Students questioned if what they wondered was right or wrong.

Implications for teaching

The team felt starting with a visual representation with no symbols and asking students what they notice and wonder gives students an entry point, and allows teachers to use those noticings as a verbal representation of important relationships in the task. However, the visual representation needs to consider recent experiences students have had that illustrates ideas connected to the mathematical goal. The more often students are asked what they notice and wonder, the more productive those times will be.

Suppressing Generalizations

Suppressing formal generalizations that students may have been previously told from coming out in the whole group discussions until later in the lesson.

Observations

- Only one student mentioned negative reciprocals early in the lesson.
- Other students argued with that student in small group but were not apparently influenced by the comment.
- Many students said, “The slopes are opposite” early in the lesson.
- Students did not seem to have a deep enough understanding of slope to connect that understanding to slopes with negative reciprocals.

Implications for teaching

It seemed that suppressing formal generalizations that students make in the whole class is important when the teacher anticipates the students could develop that generalization on their own with understanding.

Encouraging Generalizations

Asking students to generalize what they see happening in words before producing any more formal representations like tables, graphs and equations.

Observations

- Students seemed to want to write down what they were thinking.
- Some students were trying to write down equations that did not directly relate to the situation.
- Students’ verbal statements were more accurate than their symbolic representations.

Implications for teaching

Asking students to write a verbal representation before symbolic representations allows them to take time to make sense of their ideas in what they see is happening before producing symbolic representations without meaning.

Think-Pair-Share

Embedding multiple opportunities to think-pair-share.

Observations

- When students were asked to connect representations in small groups they were

discussing how those representations were similar, increasing their depth of understanding of slope and perpendicular lines.

Implications for teaching

As a result of this action...

Students press each other to think more deeply about what each other is thinking.

Students who might not have something to share in the whole group get an opportunity to listen to the ideas of each other in their small groups. Those who are more advanced also get the opportunity to articulate their thinking so they can understand the concepts more deeply. Hearing other students' perspective supports students in connecting between representations.

Choosing Students to Share

Asking specific students to share with the whole group something that was said in the small group discussions

Observations

- Many students are resistant to sharing their ideas with the whole group.
- When one student shared her way of thinking with the whole group, others had opportunity to see a different perspective.

Implications for teaching

This strategy allows the teacher have some control over what ideas come out in the whole group for others to think about, while still allowing the ideas to come from the students. Additionally, the student who is going to share gets a chance to think about what is going to be said. This could allow other students to continue to focus without fear of being called on.

Trying Other's Way of Thinking

After discussions about similarities among representations have been discussed in small groups and the whole group, having students try other students' ways of thinking.

Observations

- Students were asked to create a physical representation from the table.
- Students were struggling, but trying to support each other.
Students were critiquing each other when trying to produce a representation, using the representations to justify their critique.

Implications for teaching

This action seemed to support deepening students' understanding. This portion of the lesson made visible the idea that students who are quick to get correct answers did not have a deep understanding of the concepts any more than those who are less quick to get right answers.

Analyzing Similarities Between Representations

Explicitly asking students to discuss similarities between representations and where they see each part of a representation in a different type of representation (“Where do you see each part of the statement in the visual?” etc).

Observations

- Students had a difficult time deciphering what it means to see each part of a representation in another representation.

Implications for teaching

When students can articulate similarities in parts of representations with parts of another representation, they start to understand the connections and have multiple problem-solving access points. Therefore, teachers need to be regularly and explicitly asking students to compare representations.

Lesson Study – Visual Patterns

November 2, 2017

Research Question:

How can teachers support productive struggle for all students?



This lesson study team hypothesized that the following actions would support students in productive struggle and SMP 1 (making sense of problems and persevering in solving them). Below are each of those actions along with observations made regarding each action and implications for teaching.

Visuals

Beginning the task with a visual and asking students how they see it, and then asking them to see if they can see it differently before they hear any other students' thoughts

Observations – Initially all students seemed highly focused on the task to start, providing a wide variety of responses. Providing the time for students to think on their own allowed for the variety of ways of thinking. The teacher pressed the students to explain their ways of thinking in the whole group to get more students to understand what they meant.

Implications for teaching – Making students fully explain what they're trying to say supports productive struggle by allowing others to understand different ways of thinking at the beginning of the task. Hearing others' different ways of thinking can deepen our own understanding. It seemed that this action would often support production.

Patterns

Asking students to describe how they see the image (pattern) growing, sketching what they think another figure in the pattern will look like and explaining why

Observations – Students had difficulty trying to produce what another student was thinking. The teacher also mentioned that it was difficult to interpret students' representations in the moment.

Implications for teaching – Sequencing the representations in a way that students can describe what is similar about them supports sense-making and productive struggle.

Think-Pair-Share

Embedding multiple opportunities to think-pair-share

Observations – Students had many opportunities to think on their own first, then share in thirds. Students who were showing they wanted to quit, stopped talking for a minute or two but it seemed that noticing others' still talking engaged them back into the conversation.

Implications for teaching – Allowing time for students to think on their own first delays instant gratification while still letting students who need more time to develop their own ideas, and simultaneously allowing students who are quick to answer to reflect on their own ideas at a deeper level. Getting support from their peers certainly contributed to productive struggle.

Visual Representations

Displaying the actual representations that students produce (rather than writing or copying what a student says on the board)

Observations – Students could see that their own ideas are driving the learning and the conversation.

Implications for teaching – This allowed students to focus on one idea at a time, rather than a wide variety of ideas or concepts (through multiple problems being displayed) and have in-depth conversations on one idea. This also seemed to show that teachers value what students do and think.

Verbal Representations

Asking students to describe what another student is thinking through producing a representation that would show the same idea

Observations – Students were testing out each other's ideas and pressing each other for precision in their wording and their representing.

Implications for teaching – This action allowed the students to see that they have agency and authority over the mathematics, and they are the ones who decide whether something is mathematically accurate or not.

Symbolic Representations

Asking students to produce symbolic representations after they've had multiple opportunities to produce visual and verbal representations

Observations – By the time we asked for equations (after about 1 hour of visual and verbal representations), 3 students produced algebraic equations. Once students had a rough draft equation, they worked to prove whether that equation matched the visual representation.

Implications for teaching – The team felt that this action validates productive struggle. This also allowed students to slowly refine their ideas and representations, comparing their symbolic representations to the visual representations.

Lesson Study – Equivalent Expressions

November 7, 2017

Research Question:

How can teachers pose purposeful questions to facilitate meaningful discourse in supporting productive struggle for all students?



This lesson study team hypothesized that the following actions/decisions are important in posing purposeful questions to facilitate meaningful discourse in supporting productive struggle for all students.

Think-Write-Pair-Share

Embedding multiple opportunities for students to think-write-pair-share will promote more disagreement amongst students (discourse).

Observations – It seemed that many students were considering other students' perspectives in the small group discussions without arguing. There was some, but limited, challenging of each other's reasoning. There also seemed to be a significant percent of students that engaged in discourse in small groups, as well as writing down their thoughts. Some students were encouraging other students in their groups to share their ideas.

Implications for teaching – The team felt using think-write-pair-share on a regular basis in classroom routines would allow students to become comfortable with that structure, and ultimately more efficient in how the time is used. The team believes that embedding multiple opportunities for think-write-pair-share would also support

productive struggle because it prompts students to use their current understanding in a small group setting with less fear of being “wrong” in front of a large group.

Launching the task

Limiting what students see in the launch of the task will allow for revision of initial thinking (productive struggle).

Observations – Several students did revise their thinking when the second expression was introduced. Some students did try to use the introduction of the second expression to support their initial thought (rather than revise). One student had a correct response after the first expression was introduced, but revised to an incorrect response when the second expression was introduced.

Implications for teaching – Encouraging students to understand that it is productive to revise their thinking may require ongoing, explicit discussion about the value and expectation to revise their thinking as a result of new information introduced in a task or through discourse. The team believes that strategically limiting what is introduced in the task so that there are opportunities for students to revise their thinking is a strategy that would promote productive struggle over time. This would also send the message that being wrong is a part of learning whereby enhancing productive struggle over time.

Assessing and advancing reasoning

Thinking about purposeful questions in terms of assessing reasoning and advancing reasoning will enhance the teacher’s capacity to facilitate meaningful discourse.

Observations – Most of the questions in the beginning of the lesson were “assessing” questions. As the lesson progressed, the questions took on more characteristics of “advancing” reasoning questions. While some questions may have qualified as having both purposes, the most apparent question that seemed to advance reasoning had to do with identifying specifically where each part of the expression matched the tape diagrams.

Implications for teaching – Thinking about purposeful questions in terms of assessing reasoning and advancing reasoning supports the teacher’s crafting of questions throughout a lesson. Teachers should be asking questions to assess student reasoning, and then questions that advance students’ reasoning based on responses to the assessing questions. Students can be advancing their reasoning without having full awareness to the depth they are thinking and it seems to be wise to explicitly state when that is occurring in the classroom.

Anticipating student thinking

Teachers anticipating student thinking collaboratively and what questions to ask to make progress towards the goal of the lesson before the lesson will support the posing of purposeful questions.

Observations – The team successfully anticipated many of the representations, ways of reasoning and behaviors that students actually did in the lesson. This allowed for much thought to be put into how the teacher can respond to different ways of thinking with assessing and advancing questions.

Implications for teaching – In order to pose purposeful questions that assess and advance reasoning, teachers should as much as possible, anticipate student representations and ways of thinking in a collaborative setting. Having desired representations available for teachers' use during the lesson is a product of collaboratively anticipating student representations and thinking.

Connecting representations

Explicitly asking students to connect visual representations with symbolic representations will support meaningful discourse and productive struggle.

Observations – Students initially had an opportunity to interpret what the tape diagram was showing. Interpreting the tape diagrams without the expressions seemed more difficult for students relative to when there were expressions to try to match the visuals to. The discourse seemed richer when students were connecting visual and symbolic representations. Focusing on expressions before equations drew attention on the meaning of the representations instead of trying to “get an answer”.

Implications for teaching – Explicitly asking students to connect visual representations with symbolic representations will support meaningful discourse and productive struggle. However, the questions that are asked of students need to be purposeful (whether assessing reasoning or advancing reasoning). The visual representations shed light on the meanings that underlie the symbols in the expressions so designing questions to illustrate the connections between those representations would make discourse meaningful and struggle productive.

Numberless Word Problem Lesson Study

November 8, 2017

Research Question:

How can teachers use and connect mathematical representations to advance students' reasoning in solving unknown addend problems?



This lesson study team hypothesized that the following actions/decisions are important in how teachers can use and connect mathematical representations to advance students' reasoning in solving unknown addend problems. Each hypothesized action is italicized, followed with observations from the lesson study team and implications for teaching and/or learning that extend beyond this lesson to teaching in general.

Length context

1. *Using a context that includes length measurements would likely prompt students to use length models (number lines) to represent their thinking.*

Observations – Despite the context using miles, students still seemed to represent in “laps” or discrete objects. About 25% of the students represented with a number line. The context did not seem to impact the representations students chose to use. They generally just looked at the numbers and used a tool or representation that they felt most comfortable with for those numbers. About 40% of students used a visual of the base ten blocks, with a fairly even distribution of 4 other representations used including physical base ten blocks, algebraic equations, number bonds and numeric representations.

Implications for teaching – Finding ways to integrate the number line in other tasks may be important for students to become more comfortable in choosing to use it. Having a context that includes length units is not enough for students to represent with a length model. It seemed that having contexts with consecutive actions by the same subject (same person going a distance one day, then more on another day) would be more likely to be represented with a number line than having two different subjects in the story problem (Mrs. Dent’s class and Mrs. Pfefferkorn’s class).

Launching the context without numbers

2. *Using a numberless word problem will focus students on the operations and relationships more than “what to do”.*

Observations – There were no students just putting the numbers together without thinking about their meanings. Students were thinking about different combinations that could work in the situation (“it could be 90 and 10, 75 and 25, or 50 and 50”, etc). Students also expressed interest in knowing more information so they could figure out which class ran more miles.

Implications for teaching – This structure allows students to slow their thinking down, make sense of the quantities and relationships in the context and not rush into answer-getting. It was clear that this type of task focuses students on the operations and relationships between the quantities more than what procedure to do or use. Making the context numberless and not including a “question” to answer also supported students in developing the question for the context. The team also believes numberless word problems will likely support students in thinking of variables as quantities that can change, not just unknowns or missing value.

Multiple opportunities for focused discourse

3. *Incorporating many think-pair-share opportunities will allow for more opportunities for students to internalize connections among and between representations.*

Observations – Telling students to be prepared to share out what their partner was thinking before they did a “turn and talk” helped students to focus on what others were thinking not just their own thoughts. This prompted students to ask each other what they thoughts in small group discussion. Students were clearing up misunderstandings about the context in these pair-shares, and all students had an opportunity to verbalize the connections they were making, and compare how they were thinking about the connections between representations to how their partner was.

Implications for teaching – These opportunities allow students to clear up misunderstandings or misconceptions in their small groups. Having students think-pair-share during the connecting representations phase of the lesson holds students accountable for connecting representation (rather than just listening to one student’s verbal description of the connection).

Connecting between representations

4. *Encouraging students to change the directionality of connections between representations will promote deeper understanding of connecting subtraction with unknown addend situations.*

Observations – Some students were talking about similarities and differences between representations without prompting. There were numerous opportunities for students to make connections between various types of representations (contextual to verbal, contextual to visual and visual to contextual, visual to symbolic and contextual, etc). The overwhelming majority of the lesson included these opportunities, not only in the concluding discussion.

Implications for teaching – Explicit questions that ask students to explain where they see one representation in another is crucial for students to change directionality and promote depth. Encouraging students to change the directionality of connections between representations will promote deeper understanding.

Sequencing representations

5. *Sequencing the representations for students to connect between from the most commonly used representation (visual of base ten blocks) to other representations, and then focusing the connections on other representations will support “advancing their reasoning” beyond the visual of base ten blocks.*

Observations – The team noticed that introducing the symbolic representation with the unknowns was effective, as students were able to make early connections between the visual representations and the equations, but then continue to do so with other visuals throughout the summary discussion, including with the number line.

Implications for teaching – The team agreed that this sequence was effective in this lesson. Introducing the symbolic representation early in the sequence allows for changing directionality of the connections between representations, as the teacher can continue to ask students to connect other representations to the symbolic representation. A potential follow-up task might be to have students use the number with different numbers/context to solve a similar problem.

“Bansho” (board work organization)

6. *Having the teacher select specific representations in the above-mentioned sequence where those students transfer their representations to specific locations on the board prior to the whole group discussion will allow students to see the mathematical storyline involved in viewing subtraction as an unknown addend situation.*

Observations – As some students were finishing their representations, the selected students displayed their representations in the specific place the teacher asked them to. Therefore, the whole class summary discussion focused on the representations began with all students being able to see all of the representations that were going to be discussed in a sequence from the left side of the board to the right.

Implications for teaching - Having students put their representations on the board at the beginning of the whole group discussion to connect representations allowed for a smooth transition between each discussion about the representations, and allowed students with varying levels of understanding to remain engaged in the discussion and connecting of representations. For example, some students finished discussing what they were asked to discuss, and then began to discuss connections to other representations that were forthcoming. The team felt this was a highly effective strategy to promote connections between representations for all students.

Individual Team Member Take-Aways

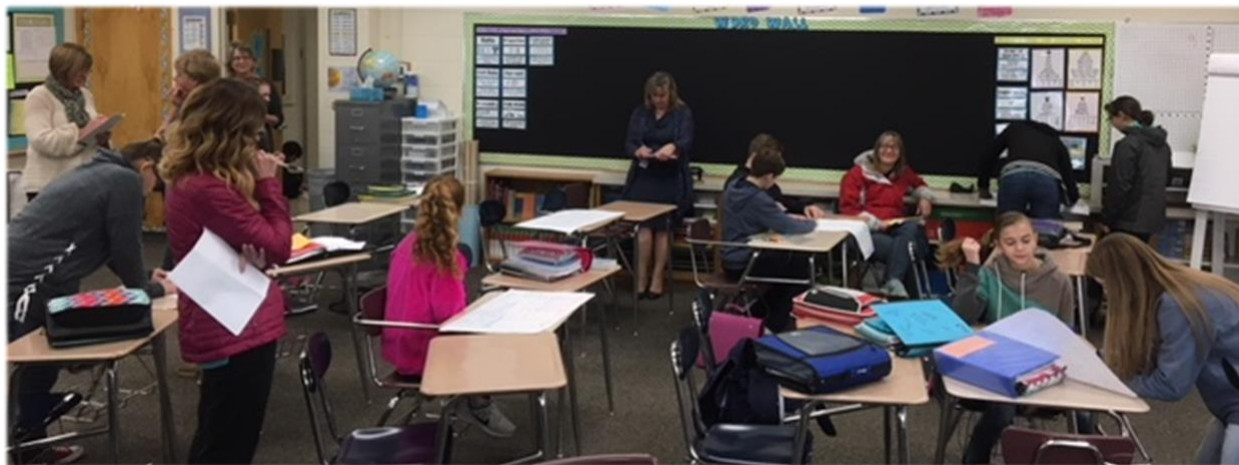
- “I want to continue using the board-plan displaying all of the representations at once, rather than in pieces is something that I want to do more of.”
- “Regularly using the whiteboard as a display of what the students are thinking during the whole group connecting representations is my next step.”
- “I need to do more think-pair-share in lessons, and maybe need to make physical changes to the classroom.”
- “I like the idea of using similar context to follow-up but changing the numbers or asking for a modification of a representation that was used in the previous day’s problem. For example, starting the next lesson with a student’s representation from this lesson, but extending that thinking into a new context.”
- “I think our materials may be moving too fast. I would like to think about slowing down to go faster, going in-depth in one problem in one day.”
- “I would like to do more of a slow-reveal of tasks (withholding the numbers and the question), with think-pair-shares in between each part of revealing new information like another number, the question, etc.”

Unit Rate Lesson Study

November 28, 2017

Research Question:

How can teachers pose purposeful questions in supporting students in making sense of problems and persevering in solving them?



The lesson study team has hypothesized that the following actions will be important elements in posing purposeful questions to support students in making sense of problems and persevering in solving them.

1. Launch a visual task

Launching the task with a visual (video clip) and asking students what they notice and wonder will allow for high levels of entry into the task.

Observations – The task allowed for multiple entry points because there were a lot of different models that students produced without prompting by the teacher. By allowing students to produce representations together, students who had difficulty producing their own representation were still engaged in quality discussion about important mathematical ideas. Every student was engaged in starting the task and every student produced something on their own paper, and wanted to stay engaged in their groups. Students who finished early tried to produce other representations that matched their initial representations. Students were connecting their representations to the context, without prompting from the teacher.

Implications for teaching – Asking students what they notice and wonder seems to be a non-threatening way to begin a task. Having a visual task to start with likely provides a greater level of desire for students to work on the task. Giving students the direction to

create a representation however they want (without parameters) compared to telling them a specific representation to use (table, graph, equation, etc) should allow for multiple entry points. Tasks should also create opportunities for students to solve in different ways. If the task is asking for something too specific, it is unlikely to get multiple representations.

2. Multiple opportunities for small group discourse

Embedding multiple opportunities for students to think-pair-share will support their willingness to grapple with both question types (assessing reasoning and advancing reasoning questions).

Observations – Some students started by sharing a lot of ideas in the beginning of the task, but seemed to become less interested throughout the lesson. As the advancing reasoning questions became more prominent, discussion became more difficult for students. In small groups, students were rephrasing their ideas when another student in their group did not understand what they were saying. Several students were persistent in changing their ways of communicating to get other students to understand their ideas. Students were debating different mathematical ideas in small group discussions.

Implications for teaching – Small group discussions are valuable for building and deepening shared understandings. These small group discussions allow opportunities to re-phrase their words and ideas. The small group discussions allow every student to speak (have an equal voice) in responding to both assessing and advancing questions. This also re-positions students as authors of ideas instead of relying on the teacher to verify if something is correct or not. When moving from assessing questions to advancing questions, the ability to share with more than one pair is important for enhancing the conversation. Small group discussions also allow for more discourse for students who are less comfortable in sharing with the whole class, keeping students engaged in the mathematical discussion.

3. “Assessing reasoning” followed by “advancing reasoning” questions

Asking questions throughout the task to assess reasoning based on their representations and then advancing reasoning questions will support SMP 1.

Observations – Students were expected to share what they understand and explain to others. Students were asked to explain relationships between representations (alike and different). The questions that were assessing reasoning occurred mostly in the beginning of the lesson, with the advancing reasoning questions happening towards the end. The students were using mostly informal language around unit rate early in the lesson, and more formal language towards the end of the lesson. For example, students were using ratio and unit rate interchangeably in the beginning of the lesson, but deciphering between unit rate and ratio towards the end of the lesson. The great majority of students,

despite the increasing difficulty in questions, continued to try to make sense of the representations throughout the whole group discussion.

Implications for teaching – It may be important to consider the amount of wait time after questions are asked, and after students respond. The order of assessing reasoning questions before advancing reasoning questions is important because we need to know their level of understanding prior to pushing them to deeper understanding. For some students the assessing questions will advance their reasoning. By clarifying what they are thinking, students have opportunities to advance their reasoning by comparing their representations with others. It seems important to ask questions based on the students' representations.

4. Organized board of students' representations

Building the whiteboard in the whole class discussion with students' representations will allow for all students to engage with both assessing and advancing reasoning questions.

Observations – Students did not “present” their ideas. Students were transferring ideas and making connections between representations. Several students mentioned that a displayed representation was similar to their own, without being prompted to make connections to their own representations.

Implications for teaching – Building the whiteboard allows teachers to focus discussions on key feature of particular representations. Having all of the representations to be used in the whole group discussion visible allows multiple entry points through students' thinking. Co-developing the whiteboard with students' representations holds students' interest and supports making sense of problems and persevering in solving them. The sequence chosen by the teacher to display on the board can support the progression of their level of understanding.

Individual team member take-aways

- I can still direct by the types of questions I can ask. By sequencing questions ahead of time we can advance reasoning without funneling.
- Wait time after students respond is important to consider before asking if anything has anybody to add.
- We may have to be very clear to not respond before a particular student has time to respond.
- I love these types of tasks because they provide multiple entry points.
- Understanding of where students are going (lesson goal) is important in deciding what questions to ask.
- I appreciate the process that you can have multiple levels of content knowledge amongst the lesson study team and come up with a solid lesson that allows entry for all students.

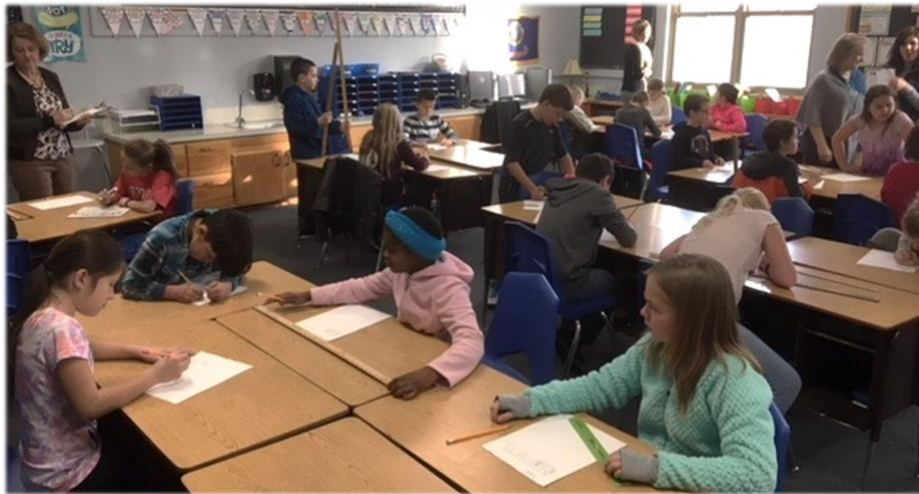
- Focusing on assessing questions and advancing questions should lead to a focusing (not funneling) pattern of questions.
- This has made me rethink in setting up a task do I have multiple entry points built in or not, and to think more about questioning patterns. Also, to focus on the structure of the lesson more, and reflect on lessons that I've done in the past.
- The time spent on anticipated responses is something we could spend more time doing.
- Splitting up the lesson over 2 days would help to support a more intentional sequence.

Measurement Conversions Lesson Study

November 28, 2017

Research Question:

How can teachers pose purposeful questions to effectively promote new learning?



This lesson study team hypothesized that the following decisions and actions would allow for teachers to pose purposeful questions to effectively promote new learning. Each hypothesis statement is followed by observations from the team and implications for teaching beyond this lesson.

1. Launch a context with the numbers removed

Using a numberless word problem will focus students on relationships between the units.

Observations – Many students initially said in partners that they can't solve the because there are no numbers. Students also mentioned that the student measured in yards would be taller than the other students. One student said "yards are way bigger than feet and inches combined." Students did not jump to calculations even after they were given numbers. It was also noticed that students became a little more anxious when the numbers were introduced, compared to when the numbers were absent. In summary, students paid attention to the comparison of units.

Implications for teaching – Starting with a numberless problem, allows multiple entry points into the task regardless of students' current level of understanding regardless of content, helps students to focus on the units, ideas and relationships, and levels the playing field at the start of the task for students who are less procedurally fluent.

2. Multiple opportunities for small group discourse

Incorporating many think-pair-share opportunities will allow opportunities for more students to think about and respond to both assessing and advancing reasoning questions.

Observations – Students clearly listened to each other better during think-pair-share discussions than during whole group discussions. Students did share genuine ideas with each other in think-pair-share discussions. It was noticed that many students did listen to each other well, possibly more than average, rather than just speaking to each other at the same time. Students did talk in small groups about what they were asked to talk about. The teacher's positioning around the room seemed to support students' willingness to struggle and problem-solve.

Implications for teaching – Students need to be given daily and multiple (per day) opportunities to listen to others' thinking and share their own thinking to reinforce ideas, clarify their own thinking, rehearse for whole-class discussions, attend to precision and revise their own thinking. Not only does this allow time for students to talk, but it provides an opportunity for the teacher to formatively assess what students are thinking, and make decisions about where the whole group discussion goes and which students can help move the discussion in a productive direction.

3. Double number line

Spending time prior to this lesson using the double number line with familiar contexts (cents and nickels) will promote the use of this representation by other students. Building the double number line with the whole class during the research lesson will allow opportunities to advance students' reasoning in describing the relationships between inches, feet and yards.

Observations – No students used a double number line to represent the problem on their own. During the whole group discussion at the end of the lesson, many students were able to explain the correspondence between inches and feet on the number line. Students were trying to describe equivalence in the context of the double number line.

Implications for teaching – Two days is not enough time with a new representation for students to choose to use it without prompting. Spending time with double number lines outside of the measurement context should not interfere with many students' capacity to make sense of the relationship between inches and feet on that representation. This particular representation would likely support students' thinking during the whole group discussion and building of the double number line. Students will need multiple opportunities using the double number line in multiple contexts before they choose to use it.

4. Questions that assess reasoning then advance reasoning

Asking questions throughout the task to assess reasoning based on students' representations and then advancing reasoning questions will support new learning.

Observations – One student had a physical representation involving cm and inches. The teacher asked several assessing questions, then an advancing reasoning question. The teacher then walked away, and the student continued to try to revise his thinking on his own, moving in a more productive direction.

Implications for teaching – It may be important to explicitly tell students that the teacher is going to walk away after an advancing reasoning question because he/she believes that the students can make progress on their own. Posing purposeful questions has to do with asking about thinking and reasoning, not just an answer, and asking students questions that we do not know the answer to. Defining the mathematical goal ahead of time is crucial in developing questions that can advance reasoning. There is a significant difference between asking questions that are focused on getting an answer to the task compared to purposeful questions that advance reasoning towards the mathematical goal. It is essential to ask assessing questions before advancing questions to find out what students know that determines what would be advanced reasoning for particular students. When teachers begin with a task that allows for multiple entry points like a numberless word problem, differentiation can occur for all students based on the questions that are asked that both assess and advance students' reasoning.

Systems of Linear Equations Lesson Study

December 5, 2017

Research Question:

How can teachers effectively pose purposeful questions to support students in making sense of problems and persevering in solving them?



This lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to support students in making sense of problems and persevering in solving them.

1. Launch a context with incomplete information

Launching the task with incomplete information will allow students to see the need for more constraints, ask their own questions.

Observations/Evidence – The students in this class asked for more information after about 2-3 minutes of trying to solve without the constraints. When students had the information they needed, they struggled producing representations. Some students needed some urging from the teacher to produce an additional representation beyond their first.

Implications for teaching – It seemed that consideration may be needed for some students who are less confident in their problem-solving as they may be less likely to protest that more information is needed. While the task was launched the day before the observation of this lesson, the team still felt that this is a teaching practice to consider in the future.

2. “Assessing reasoning” followed by “advancing reasoning” questions

Asking questions throughout the task to assess reasoning based on students’ representations and then advancing reasoning questions will support SMP 1.

Observations/Evidence – One group was observed that when the teacher came around during the small group time, the questions seemed to be about progress being made rather than reasoning. It seemed that some students were responding to the assessing questions in think-pair-share moments and advancing their reasoning at the same time, but then struggled to respond to the advancing question in a way that was different than the assessing question responses.

Implications for teaching – Teachers should ask assessing and advancing reasoning questions in the whole group discussion, but condensing the questions into fewer and more powerful is important to consider. Students can present their way of thinking in place of assessing reasoning questions in the whole group discussion, and the teacher can follow-up with advancing reasoning questions. Advancing reasoning questions should be thought about ahead of time, but also flexible enough to respond to how students are thinking.

3. Grouping students by similar ways of thinking

Strategically grouping students by similar ways of thinking to co-produce representations after their independent work for whole group discussion will strengthen the representations produced and allow for greater access to advancing reasoning towards the mathematical goal.

Observations/Evidence – The teacher noticed it was difficult to determine her role during this portion of the lesson, while the observing teachers also recognized that students were not necessarily advancing their reasoning during this time. Students were trying to make sense of each other’s thinking during this work time and asking each other questions about their different ways of thinking. The team noticed some students attempted to verbalize their thinking in their groups, but struggled to do so, many were trying to make sense of the quantities relative to the context, and many students were also justifying their thinking in these small groups while co-producing a common representation. Some groups had a dominant member of the group who seemed to determine or influence the decision as to whose representation to re-produce onto the larger chart paper.

Implications for teaching – This could be a time (after students work independently, but then work together to produce a group representation) where the teacher continues to ask assessing and advancing reasoning questions of the small groups as they co-develop representations for the whole group discussion. Two questions that could be asked during this time is “Why do you think you are grouped together like you are?” or “What are the

similarities in the ways of thinking that you used to solve the problem?” It is also important to consider giving students a moment to review their own thinking from the previous day before re-engaging in dialogue.

4. Small group discourse

Embedding multiple think-pair opportunities followed by think-pair-share will allow for opportunities to make connections between the questions being asked.

Observations/Evidence – Some students were not making sense of the conversation in their small groups, but during the whole group discussion about those questions they showed evidence of making meaning of the intent of the question asked.

Implications for teaching – It is certainly important to allow many opportunities for students to discuss their responses to both assessing and advancing reasoning questions, and for time to think about their thoughts before they discussed them in those small groups.

5. Organized board of students’ representations

Building the whiteboard in the whole class discussion with students’ representations will allow for all students to engage with both assessing and advancing reasoning questions.

Observations – Some students were less confident in their own representations, but when they saw others make meaning from their representations their opinion of their own representation was more positive, and they also considered other ways of thinking when they saw further representations.

Implications for teaching – Comparing different ways of thinking as seen on the board helps strengthen connections that students should be making and for students to see different ways of thinking that they did not come up with on their own. Having all of the representations on the board at the same time from the beginning of the whole group discussion could be considered to make connections across representations and allow the teacher to ask a lower number of questions, potentially allowing for higher levels of engagement in the discussion.

Individual team member take-aways

- Displaying students work in an organized way on the board.
- Interested in the idea of collecting the students work before hand to think about the sequencing.
- Splitting up a lesson over 2 days even with a block schedule.
- Grouping students based on their ways of thinking when we want to have them co-produce a representation.
- I want to really listen in on my students’ small group discussions to figure out to what extent are they really engaging with each other.

- I need to make sure that I'm asking both assessing and advancing reasoning questions as I might tend to assume that I understand what they understand before I ask an advancing question.
- I'll try to continue planning questions before a lesson occurs.

Systems of Linear Equations Lesson Study

December 12, 2017

Research Question:

How can teachers effectively pose purposeful questions to advance students' reasoning?



This lesson study team has hypothesized that the following actions will be important elements in posing purposeful questions to advance students' reasoning. Those hypotheses are below, followed by the observations and evidence collected pertaining to them by the lesson study team, and implications for teaching beyond this lesson regarding those teaching actions.

1. Launch a task with incomplete information

Withholding some portions of the context will allow students to ask more questions, resist jumping into a solution attempt, and gain buy-in to the context, while allowing the teacher to assess what students understand about slope and y-intercept in a context.

Observations – Many students were saying they needed to know the cost of the shoe rental to figure it out right away and that it depends on the number of games played. Some students wrote algebraic equations in slope-intercept form, but slope and y-intercept were not explicitly discussed by the students in their individual representations.

Implications for teaching – Withholding information from the context at the launch of the task allows students to focus on the meaning and relationships within the context instead of the trying to get an answer. By not providing all of the information in a contextual problem in the launch of the task, this team feels that students may have more

opportunities to discuss the context without trying to solve the problem, which allows for more students to have a more equal entry point into representing and solving the problem.

2. “Assessing reasoning” followed by “advancing reasoning” questions

Asking assessing reasoning questions will make student thinking visible, and allow students to reflect and give justifications of their own thinking. By posing advancing reasoning questions after assessing reasoning questions, we are deepening students’ understanding from their current understanding prior to and during the whole group discussion.

Observations – The teacher was able to ask assessing reasoning questions by moving in a rotation around the room during small group discussions and asking students to make sense of their representation approximately every 4 minutes. Students had opportunities in small group discussion to talk about how their graphs connected to the context. When the teacher asked how the lines represent the data in the table, students described how “one bowling alley starts out cheaper, the costs were then the same, and later it switched”. In one group, after the teacher asked if they can use the equations they produced to show when the cost is the same, students began to argue over the visual aspects of the representation. Students needed little direction on making connections between representations in the whole group discussion and could see and describe how the students’ representations connected. One group of two students who wrote equations during the launch of the task engaged in very limited discussion. The teacher repeatedly asked students in small groups how their representations showed which is a better deal.

Implications for teaching – Asking assessing reasoning questions gives more opportunities for discourse between students, and forces a deeper analysis of what they wrote down. These questions also allow the teacher to decide when students are ready to advance their reasoning or if they need further reflection on what they have already done. Assessing questions in small groups before the whole class discussion also helps the teacher understand what students understand on their own. Teachers should be aware of and allow for the necessary time to assess and advance students’ reasoning. Questions that advance students’ reasoning will students make connections between representations in the whole group discussion, thereby promoting depth of understanding.

3. Walking away after asking “advancing reasoning” questions

By walking away from students after asking advancing reasoning questions, students will be more likely to persevere and continue to advance their own reasoning.

Observations – All students persevered through the task during the small group discussions and the whole group discussion. No students in this lesson gave up in either trying to solve or represent the problem. When the teacher approached small groups of students, students clearly turned their attention to the teacher. When the teacher walked away, students turned their attention towards each other (instead of the teacher). The advancing reasoning question was the starting point of each small group conversation for

the teacher to leave the group to think about. Students commented on how they knew the teacher would come back around to their group in a short period of time.

Implications for teaching – Asking assessing reasoning questions, then walking away from the group allows the teacher to get a sense of what students are thinking generally as a class. When this becomes a regular occurrence in the classroom, students know the teacher will come back to their group after a few minutes. Students will turn their attention to each other when the teacher walks away, so by walking away from small groups of students after an advancing reasoning question, students will depend on each other to do the mathematical thinking rather than depending on the teacher to answer their questions.

4. Grouping students by similar ways of thinking

Strategically grouping students by similar ways of thinking to co-produce representations that illustrates those similarities after their independent work for whole group discussion will strengthen the representations produced and allow for greater access to advancing reasoning towards the mathematical goal.

Observations – When students were asked to discuss why they think they were paired together after producing their own representations independently, many students compared their different ways of thinking and found similarities and differences. Asking students to figure out why they were grouped the way they were, prompted students to compare their representations at a significant level of depth. They inherently looked for similarities and differences in their own representations. This decision to group students this way also led students to produce a representation that the lesson study team anticipated using in the whole group discussion. Because the representations in these small groups were similar, less time was spent by students on deciding on a representation to “copy” onto chart paper. Since their reasoning was similar in these smaller groups, less assessing reasoning questions were needed to be asked to get each student in the group on the same level and ready for an advancing reasoning question.

Implications for teaching – Teachers should consider launching conceptual tasks to students on one day to give time to look more in-depth into the representations that they produced. Teachers should also consider grouping students by similar ways of thinking and having them discover what those similarities are. By using both of these actions, students can study each other’s thinking in greater depth, and contributing to the development of co-developed representation that benefits the entire group. This also better prepares students for access to more advanced reasoning during the whole group discussion when comparing other representations and ways of thinking.

Individual team member reflections

- I definitely want to try strategic grouping by similar ways of thinking and continue thinking about its connection to providing greater access for all students.

- I realize the difference between asking kids to discuss their ways of thinking compared to asking students to figure out why they are grouped together.
- I need to remember that good questions begin with “why”.
- I need to be focusing on asking assessing and advancing reasoning questions, but be less focused on avoiding a funneling pattern of questioning.

Reasoning with Fractions Lesson Study

January 17, 2018

Research Question:

How can teachers use and connect mathematical representations to support students in revising their own thinking and advancing their learning?



This lesson study team hypothesized that the following actions would be important elements in using and connecting mathematical representations to support students in revising their own thinking and advancing their learning. Those hypotheses are below, followed by the observations and evidence collected pertaining to them by the lesson study team, and implications for teaching beyond this lesson regarding those teaching actions.

1. Using a task with multiple entry points

Using a task with a length context but not specifying a particular type of representation will allow for multiple entry points, a variety of representations and ways of thinking.

Evidence/Observations – It seemed that most students were able to start somewhere, whether it was an area model or a length model. The representations that students produced on their own included number lines, area models (with both same size wholes, and different), bar graphs, symbolic representations with equivalent fractions and equivalent fractions on a number line. Even within similar representations, students showed several different ways of thinking that included using benchmark fractions ($\frac{3}{4}$), replacing fractions with equivalent, comparing the part to the whole and comparing same size wholes.

Implications for Teaching – Tasks must provide students with an entry point and allow for a variety of ways to attack and solve the problem. Tasks that have multiple entry points often have a context to attach meaning to quantities and should not specify which representations, tools or ways of thinking students should use. Tasks should fit the mathematical goal, not just encourage students to simply get an answer.

2. Engaging students in dialogue about explicit connections among representations

By engaging students in dialogue (small group and whole group) about explicit connections among representations, students will revise their own thinking and advance their learning.

Evidence/Observations – When students were asked to find similar ways of thinking in small groups, students had to verbalize their thinking and see how it connected with other students' in their groups. As soon as they began finding similarities, students said “I will have to re-think that”, “you could have just...”, “oh, now I see that it's ___”, “what do you mean by that”, “what are you talking about here”, “explain what you mean”, “I basically did what you did except ___”, “so I guess it would have been easier if I would have ___”, “wait, why are you doing that”, etc. Students would point out misconceptions in each others' thinking. Since the task was to produce a new representation, students were forced to make some revision and advancement in their thinking. Several students revised their thinking and advanced their learning at the end of the lesson relative to the beginning of the lesson when they produced their own representations.

Implications for Teaching – Both the small group and whole group discussions to compare students' representations are valuable in supporting students in revising their own thinking and advancing their learning. It is important for the teacher to ask students to explicitly discuss the connections among the representations, particularly what is similar in their ways of thinking. The team also agreed that articulating a clear learning goal is critical in deciding what questions to ask students and what connections to focus students on.

3. Grouping students by similar ways of thinking

Grouping students in similar ways of thinking after they have produced their own representations allows students to assess their own reasoning, clarifying their own thinking, and change directionality among representations, and these will support students in revising their thinking.

Evidence/Observations – Students started in these groups and had to clarify their thinking to the others in their small groups, and many students asked each other what they meant. As they shared what they were thinking as they produced their individual representations, they both clarified what they were thinking and found some mistakes or misconceptions in their thinking. Many students changed their models/representations. There were many instances of students revising their thinking in this part of the lesson. The teacher in this phase of the lesson was asking assessing reasoning questions and questions that ask students to compare

each other's representations and ways of thinking. The teacher seemed to not be evaluating students' thinking by saying "good" or validating one way of thinking over another, but rather rephrasing what some students said. When the teacher rephrased students' statements, it helped clarify what that student was thinking for the rest of the students in that small group.

Implications for Teaching – The team felt this action was successful in allowing students to assess their own reasoning and clarifying their own thinking. There was some, but limited changing in directionality among representations during this time. This also allows students who may not fully engage in the whole group discussion to have rich discussion before the whole group discussion to compare representations and ways of thinking. The students were productively struggling during this time, and the team feels this is supported by grouping students by similar way of thinking. The teacher's role during this time is to mainly ask assessing reasoning questions and rephrasing students' ideas, and in this case, supported students in revising their thinking and advancing their learning.

Individual take-aways

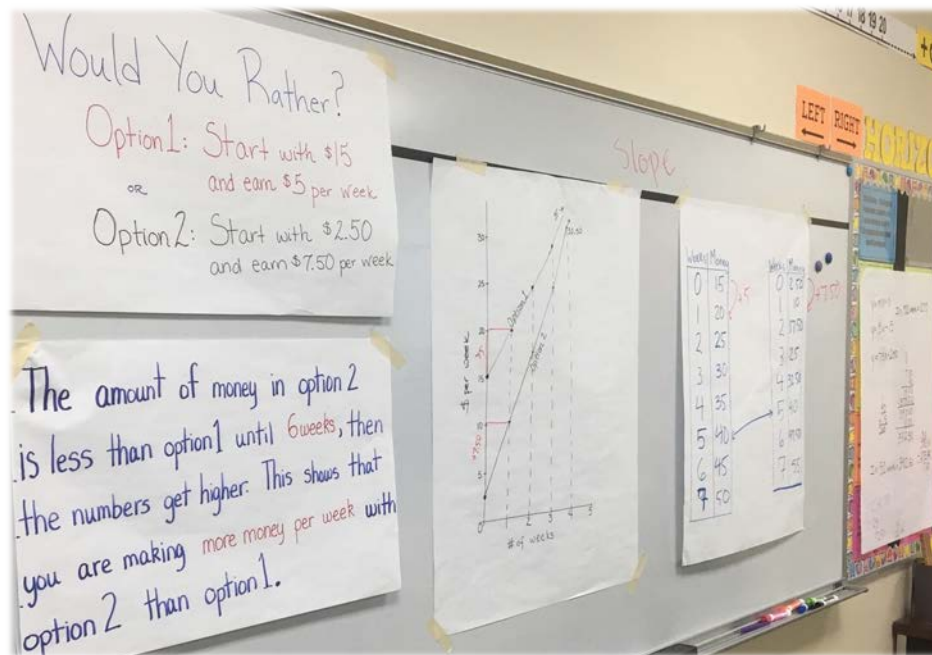
- "My whole group discussion should focus on one clear mathematical goal."
- "Make sure the task has multiple entry points by not specifying or implying which representations to use."
- "I liked organizing students in small groups to look at their representations together."
- "It's important to not judge what students understand or don't understand without asking assessing reasoning questions."
- "Sometimes we forget about pacing and it's ok to take time to plan lessons together around an essential question."

“Would you rather...” Lesson Study

February 1, 2018

Research Question:

How can teachers use and connect mathematical representations to advance students' learning of systems of linear equations?



The lesson study team hypothesized that the following actions would be important elements in using and connecting mathematical representations to advance students' learning of systems of linear equations. Each hypothesis is listed below, followed by evidence collected during the lesson study cycle and implications for teaching beyond this lesson.

1. Advancing reasoning questions during independent work time

Asking advancing reasoning questions while students are working independently will produce representations that are useful for making the underlying mathematics more visible in the whole group discussion.

Evidence/Observations – The representations students produced independently were certainly useful for making the mathematics visible during the whole group discussion. The teacher of the lesson did prompt students to produce additional representations, without generally asking for a specific type of additional representation. When students said they didn't know what other type of representation to produce, the teacher prompted students to think of other representations that were discussed briefly at the beginning of

the tasks that were options. The teacher of the lesson felt the question “Is there any relationship between the amounts for Options A and B that you think is important?” seemed to be helpful in students to advance their reasoning. “Is one option always better?” also seemed to be a question that allowed students to continue to make progress in their independent problem-solving.

Implications for teaching – Posing questions that ask students to defend their thinking seem to promote productive struggle during independent work time and simultaneously produce higher quality representations for the whole group discussion. Also asking students to try to look at it through the eyes of another person seemed to promote more clear representations.

2. Students analyzing other students’ representations

Asking students questions about each other’s representations will enhance students’ capacity to use representations as tools for problem-solving in future lessons.

Evidence/Observations – Some students readily saw that the representations were showing several similar ideas. Students seemed to be looking at each representation whether asked to or not, while others noticed differences first. Many students got out of their seats during small group discussions to reference ideas on each other’s representations. Several students began talking about their own work first, but transitioned to talking about the mathematical ideas in each other’s representations. Some students struggled with continuing the conversation the answer was established, but the continued conversation was the vehicle for students to grow in their understanding. Students were initially not engaging with the representations, but over 20 minutes or so several students made more generalizations, especially in the small group conversations. The teacher specifically selected certain students to share in the whole group.

Implications for teaching – It is important for teachers to be patient and wait for students to re-visit the mathematical ideas after they found an answer while students make connections between representations, as well as slowing the pace to allow for deeper conversations about the connections between representations that make the important mathematics visible. Students can grow beyond only sharing what their strategy was to understanding the mathematics more deeply through analyzing each other’s representations (SMP 3). If students only focus on their own representation, the important mathematics may not be as visible because of the nature of that particular representation. The team feels this teaching strategy also supports students in enhancing opportunities for students to “look for and making use of structure” (SMP 7). The representations served to support students’ thinking in responding to the higher-level questions that they were being asked to discuss.

3. Displaying all of the selected representations at the same time

Choosing to display all of the representations that were selected for the whole group discussion at the beginning of the discussion will give greater access to all students to see the underlying mathematical ideas.

Evidence/Observations – The teacher was able to ask “can you see the slope in all of these different representations?” because all of the representations were visible. Students who didn’t create a graph were able to see ordered pairs in the graph. The students treated each of the representations as equal in value for learning.

Implications for teaching – This decision seems to support the idea of making connections across representations without students’ perceiving that there was one strategy or model that was more important or valuable than the others, nor see the discussion as a progression towards the “best” representation.

4. Questions allowing students to make connections

Asking questions that make the underlying mathematics visible while connecting representations in the whole group discussion will be the primary vehicle for students to make those connections, thereby increasing their depth of understanding.

Evidence/Observations – Students extended beyond their own thinking when asked to make connections between representations. They tried to find key ideas (slope, y-intercept, ordered pairs, point of intersection) in other representations. When asked about the significance of zero in the table, they referenced the context and the graph to explain why it’s important to start with zero. The depth to which students were understanding was made evident during this time as they described the connections between different representations. Students were asked “how do you see the rate of change in these representations”, which led to a discussion about different terms for the meaning of rate and rate of change. The questions that were asked higher level types of questions, and mostly included “making the mathematics visible” questions that allow students to discuss mathematical structures and make connections among mathematical ideas and relationships. The task also did not begin with lower level questions, rather an opportunity for students to form an argument.

Implications for teaching – Drawing students’ attention to the underlying mathematical concepts or big ideas present in all of the representations that were selected helps students to understand the underlying mathematics so they can eventually generalize those ideas and apply them to future situations. This action also allows students to see that a single concept can appear differently in different representations. Attending to key mathematical features of each representation sets the stage for examining more formal ideas throughout the unit, beginning from students’ current understandings. Without the students’ representations these questions would not have been able to be asked.

Individual team member take-aways

- Drawing attention to the big ideas that underlie the mathematics in the representations should be the focus of my questions in whole-group discussion.
- The importance of seeing other students' representations as opportunities to deepen their understandings (or see something in a way they couldn't).
- Prompting students to re-visit their own representation and the representation of others helps them to more deeply understand the mathematical concepts.
- Posing advancing questions that promote productive struggle allows students to think about the underlying mathematical concepts at a much deeper level.
- The way the types of questions were asked yesterday gave representations as an anchor for the rest of the unit to connect abstract ideas to.

Counting On Lesson Study

February 7, 2018

Research Question:

How can teachers pose purposeful questions to advance students' reasoning from counting all to counting on?



This lesson study team hypothesized that the following actions would be important elements in posing purposeful questions to support students in moving from counting all to counting on. Below each hypothesis statement is the evidence collected during the lesson by the team, and what the team considers the most important implications for teaching beyond this lesson.

1. Routinely using quick-images

Starting each day with quick-images, including images of a Rekenrek, will promote counting on rather than counting all as students will not be able to rely on one-to-one tagging.

Observations/Evidence – There was a student sitting close who was trying to one-to-one tag the beads, but when covered tried to imagine the beads that were covered. A few students referenced the difference in color, and using the language of “5 and one more” or “5 and 3 more”. Students were using their fingers to match the words they said to describe how many beads they saw. Several students said they were counting. Many students were using the five-structure when saying “5 and 3 more”. There were several instances of evidence of students having a handle on hierarchical inclusion. This section

was 15 minutes, consisted of 7 different images, and were flashed for about 2 seconds each.

Implications for teaching – The Rekenrek seems to be helping students in seeing 5 (or 10) as a unit and count on from there. Flashing these as quick-images will likely cut down on one-to-one tagging. This tool also seems to be helpful in supporting students in seeing groups of tens.

2. Multiple opportunities for student discourse

Incorporating multiple opportunities for student discourse will allow students to explore different strategies and think about why they work.

Observations/Evidence – Most of the pair-shares students were on topic with what the teacher asked them to talk about. Some students seemed to start only by giving an answer to what they saw on the Rekenrek. After the teacher asked students to explain “how did you see it?” students were more inclined to explain the combinations they saw. The teacher counted down to bring students back to attention. Students are quick to agree with anything that another student says, even if they just said something different in a small group conversation.

Implications for teaching – Allowing multiple opportunities for student discourse should advance students’ reasoning as it allows them to think about how other students are thinking about a question or problem. The team thought maybe offering 3 ways to say whether students agree/disagree/unsure may be a more authentic way of seeing who really disagrees or agrees. The team also thought that having “wait time two” might help to get a more authentic response from students as to whether they agree or disagree.

3. Posing “assessing” then “advancing” reasoning questions

Students will move from “counting all” to “counting on” as the teacher asks assessing reasoning questions (connecting their models to the problem) followed by advancing reasoning questions (asking students who are counting all if they can try to see numbers inside the total).

Observations/Evidence – After posing the task, the teacher asked an assessing question to determine whether students understood the context. Students had difficulty representing the problem on the rack and the images on the paper. Students also had difficulty connecting the story to the mathematical task. The teacher walked away from students after asking advancing reasoning questions. A few students would often start with counting on, but would double-check by counting all. The most common question that students were asked was “Does your model match the story problem?”

Implications for teaching – The team wondered if students were asked to predict what was going to happen during the story if that may have supported the mathematical big

ideas in the story of equivalence and compensation. It's important to ask assessing reasoning questions, then advancing reasoning questions. It is also important to ask questions that will advance reasoning based on their current understanding, but determining what those questions are is difficult to anticipate. Anticipating students' representations and questions that would go along with them are helpful, but teachers should also be flexible in using them based on what actually happens in the lesson.

4. Waiting for students to think

Explicitly making students pause before responding and the teacher attending to waiting for 3 or more seconds after students' respond will encourage students to think about their ideas rather than guessing an answer.

Observations/Evidence – Students did wait to think about their ideas before impulsively responding with an answer. Students were more apt to say what their answer was than how they figured it out. The teacher did wait, particularly during the quick-image section and it seemed to show the students that the teacher values their thinking and that they're capable of working it out.

Implications for teaching – Explicitly pausing should develop independent problem-solvers and thinkers because their thinking is valued and they have time to pause and think. Reminding students to pause and wait before they show or tell their answer is an important element before launching a task or posing a question. Waiting should also allow more ways for students to think to add to the conversation.

5. Grouping students

Optimal mismatches of students in pairs will allow the teacher to have students collaborate to advance their reasoning together by discussing an advancing reasoning question posed by the teacher.

Observations – Students were paired randomly during quick-images and the discussions were productive in sharing their thinking and advancing their reasoning. Some students changed their minds in this setting after they heard their partner's way of thinking. One student during this time quoted another student, saying "My partner ____ said...". Students were paired based on similar anticipated strategies for the bunk bed task and the discussion seemed less productive.

Implications for teaching – The team feels that optimal mismatches (not too close in thinking, but not significantly different) would be a productive way to group students during investigations. Grouping students by perceived ability seemed to

Individual team member take-aways

- The value of the optimal mismatches in pairing students during investigations.

- Learning how to navigate the landscape to help understand where the kids are and how to advance reasoning, and maybe to add activities that will help kids fill in gaps.
- I think I need to take a breath and remember that learning is slow and that we don't have to get kids to move from one end of the landscape to the other in one day.
- The ability to ask good questions takes a long time to develop.
- I need to make sure that I'm giving students the wait time after a question, and after their response, both for myself and for them because we're not maximizing learning because I'm not thinking about it
- In planning the lesson I want to take more time to think about what their responses will be so I can have some questions ready.
- I want to think more about when optimal mismatch pairing would be most effective.

Rational Number Addition Lesson Study

February 26, 2018

Research Question:

How can teachers facilitate meaningful discourse in supporting students to make sense of problems and persevere in solving them?



This lesson study team hypothesized that the following actions would be important elements in facilitating meaningful discourse in supporting students to make sense of problems and persevere in solving them. Below each hypothesized action is the evidence collected during the lesson by the team, and what the team considers to be most the important implications for teaching beyond this lesson.

1. Linking and re-voicing

Linking (the learning community revoices the work or thinking of another student)

Observations/Evidence – Some students were naturally re-voicing each others' statements in small group discussions, which then prompted students to ask questions of each other. The teacher asked students in the whole group conversation to re-voice what another student had said. Some students did also re-voice/re-state what other students' said unprompted by the teacher. The teacher also asked another student to re-voice what a student said without the student volunteering to do so. The teacher re-voiced an imprecise argument that was connected the goal of the lesson.

Implications for teaching – Re-voicing allows for improvement on clarity, to allow more students to understand what was said, and gives them a chance to check in. Re-voicing by peers and the teacher keeps more students engaged in the conversation. Re-voicing also seems to make the math more clear to more students, and make students feel more confident about their problem-solving strategies. This practice also seems to slow the pace to allow students to reflect on their own thinking and that of their peers. Additionally, if teachers consciously wait for students to have time to think after asking questions and prior to responding to students there are more opportunities for students to solidify the ideas being discussed.

2. Pressing students to justify

Pressing students to justify, support or say more about their thinking holds students accountable to the mathematics, thereby making the math more visible.

Observations/Evidence – Students were pressing each other to justify their thinking to clarify what they mean. Students were asked to make connections between their number lines and the contexts. Some students were justifying their solutions through connecting to the number lines. In some cases, students were able to notice a misunderstanding on their own rather than the teacher telling them. When students discussed their thinking about others' representations, they began comparing that thinking to their own. Students were using the context to justify aspects of the representations. When asked the difference between two number lines, one student articulated the similarities, but also that the answers were different.

Implications for teaching – Sometimes misconceptions can be brought to light, even though students may have had a correct answer. Pressing students to clarify their own understanding, helps them reflect on their own thinking as well as helping other students to clear up their own misconceptions, or at least start noticing the misconception. Pressing students to justify their thinking needs to be part of the classroom culture that is developed over time. Teachers can push students' ideas back to the class to think about instead of the teacher saying that what was said is either "right" or "wrong" to promote students' confidence and ownership for future problem-solving. Additionally, having students turn and talk often presses more students to explain and justify their thinking, and gives more voice and authority those students who are less comfortable contributing to the whole group conversation.

3. Connecting representations

Explicitly asking students to make connections between representations deepens students' understanding of the mathematics that underlies those conversations and is a supporting element in facilitating meaningful discourse.

Observations – Students did not seem to have too much difficulty in comparing representations and seemed to feel like they could discuss whatever they thought. Some students found similarities in the connections that they were making when they discussed what they noticed. Students were agreeing with each other in small groups about the

connections they were making. It seemed like students were checking to see if the representations made sense relative to the context. Some students were making the connection between what the direction of the arrows mean relative to the context. Students were talking about the direction the arrows were pointing when connecting representations (same direction, one direction, adding on).

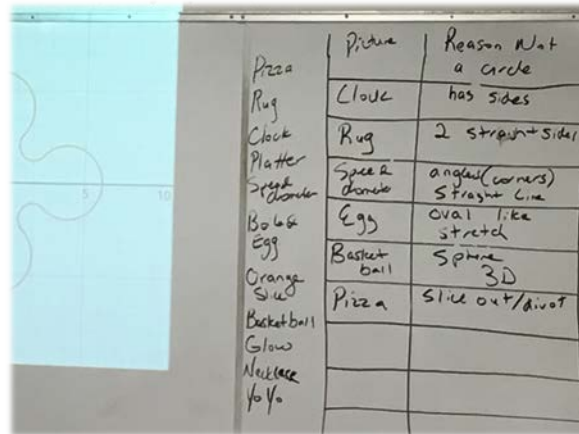
Implications for teaching – Taking time to have students discuss connections between representations might give teachers more time later because students have opportunities to clear up their misconceptions early on before they own those misconceptions. The foundational concepts will be manifested through finding similarities and differences in students' representations and asking students to make connections between representations sets the stage for ideas to refer back to later in the unit. It is also important to allow time for turn and talks when students make connections between representations.

“What is a Circle?” Lesson Study

February 28, 2018

Research Question:

How can teachers pose purposeful questions to help students view their own and others' thinking and representations as more than getting an answer, but a vehicle to learn something new?



The lesson study team hypothesized that the following actions would be important elements in addressing the research question. Each hypothesis is listed below, followed by evidence collected during the lesson study cycle and implications for teaching beyond this lesson.

1. Task with intentional disagreement

Starting with a task that lends itself naturally to disagreement may help students value the use of reasoning over answer-getting for solving future problems.

Evidence/Observations – When students were sorting and there was disagreement in small groups some students asked each other for clarification. Some items that students disagreed on were discussed, but then put to the side without consensus. When students within a small group immediately agreed there was little discussion with reasoning. Items that were not discussed and consensus gained were still being discussed at later points in the lesson even though most of the class had moved on. The teacher incorporated many turn and talks, re-voicing, having students revoice and probing thinking questions.

Implications for teaching – Opportunities for disagreement seems to encourage students to engage in the lesson more, and hence draws out more students reasoning. When students have an opinion of something, they want to argue for that position and are more likely to stay in the conversation. There may be a connection between a task that

lends itself to disagreement and student reasoning since the more robust discussion occurred, forcing students to come up with a reason to accompany their argument.

2. Finishing another students' reasoning

Asking students to make a statement without reasoning first and asking the rest of the class whether they agree or disagree allows the rest of the students an opportunity to make sense of that student's idea.

Evidence/Observations – Students had gestures and expression that showed they were trying to make sense of what another student said. Many students seemed to not be trying to make sense of why a student thought what they did, but rather they were justifying why they disagreed with what a student said. The teacher followed-up numerous times in this section of the lesson with re-voicing a students' reasoning in the whole group, and asking the rest of the class what they think that student means. Students were building off of each other's ideas during this part of the lesson, sometimes quoting each other.

Implications for teaching – It seems important for the students to have opportunities to re-voice what each other says because they become more aware of what each other is saying and thinking. Having students re-voice what each other is saying helps to validate and clarify what was said. Having to re-voice what other students said makes students think more deeply about the mathematical ideas being discussed.

3. Multiple opportunities to revise thinking

Giving students an opportunity to revise their thinking (or definitions) will communicate to students that they are not expected to know everything at the beginning of the task, but they can learn from others' ideas.

Evidence/Observations – Many students used phrases for their revised definition from what was discussed when building the table. Most students revised their definitions based on the discussions from earlier in the lesson.

Implications for teaching – It's important for students to know that they are not expected to know everything from the beginning of a lesson or unit. The students seemed to be willing to share their thinking and reasoning whether they thought they were "right" or not, so the culture of valuing communicating and reasoning seemed to be established before this lesson.

4. Use of “monsters” to contradict thinking

Intentionally introducing a misconception or “monster” will allow students to refine their thinking (SMP 6 – Attending to precision).

Evidence/Observations – Students were looking at the characteristics that they developed in the table, but saw the difference between what a circle is but that it still fits the characteristics in the table. The monster created cognitive dissonance where students thought they understood the characteristics of a circle, but it didn’t match what students knew was a circle. One student commented that you would need to “pull out” the indents. The teacher followed with an advancing reasoning question that asked “how far” it would need to get pulled out, which prompted some discussion getting closer to the idea of radius.

Implications for teaching – The team feels that introducing a “monster” would allow students to refine their thinking in other lessons as well. It may be unreasonable for teachers to create “monsters” on a regular basis, yet there is still value in introducing them whenever possible.

Individual team member take-aways

- Allowing wait time after a student responds to either the teacher or another student is powerful.
- I thought the monster was powerful because students had to evaluate the thoughts they already had come up with and reflect on whether those are still true.
- Every kid had entry into the task since there wasn’t a right or wrong answer from the beginning.
- The teacher discourse moves need to interplay with and support each other.

Decimal Fractions Lesson Study

March 18, 2018

Research Question:

What are critical aspects (pedagogical and content oriented) to support students in connecting their understanding of fractions to build an understanding of decimals?



The lesson study team hypothesized that the following actions would be critical aspects to support students in connecting their understanding of fractions to build an understanding of decimals. Each hypothesis is listed below, followed by evidence collected during the lesson study cycle and implications for teaching beyond this lesson.

1. Using fraction language to name decimals

Requiring students to say decimal values with fraction language (saying 0.6 as “six tenths”) will support their flexible use of fractions and decimals.

Evidence/Observations – The teacher used the language of fractions to describe decimals throughout the lesson, while students increasingly used fraction language to describe decimal values as a result of the teacher using that language. The teacher restated what a student said in fraction language when the student would call 0.1 “point

one". The teacher would also clarify their language as "point one" by repeating that, but then changing it to "one tenth". One student said that "point 2 and another 5 then you add them for point 2 – 5." The teacher responded by writing on the board "point 2 plus 5 is 5 point 2". The student re-phrased to say it is "point 2 plus point zero-five". Many students wrote several equivalent fractions for each of the fractions that were being discussed. Many students also showed that they had strong fraction sense.

Implications for teaching – It appeared that the students who did use fraction language to describe decimal numbers were more flexible in moving between fractions and decimals in their language. Letting students access the language from what they already understand was important, while simultaneously the teacher uses fraction language in referring to the decimal values so that students acquire that same language. Teachers should be persistent about using fraction language to describe decimals as it appears that is one way that students were using that same language.

2. Double number line

Prompting the use of the double number line to show fraction and decimal equivalents will allow students to see that fractions and decimals represent the same number with different notation.

Evidence/Observations – Students did not seem to be trying to compute/calculate the decimal value for each fraction, but rather use reasoning to determine how to write that same number in a different notation.

Implications for teaching – Since students had many experiences with fraction on the number line, the double number line will support these equivalent relationships. This model clearly shows that these various forms of numbers represent the same number because they are at the same location on the line. Having one line with fractions and decimals instead of two lines is also an important element of this model for students to see that the fractions and their equivalent decimals hold the same position on that line.

3. Beginning with benchmark fractions

Beginning the task with discussions around benchmark fractions and decimal fractions with their equivalent decimal numbers involving tenths will support the understanding that the place value position matches the denominator in decimal fractions.

Evidence/Observations – Some students were using the benchmark fractions like $\frac{1}{4}$ to help place other decimals and fractions. This also seemed to allow for an entry point into the task for students to use their foundational understandings of fractions and equivalent fractions. Some students were trying to use benchmark locations but since they didn't directly connect to tenths they reached an impasse. The benchmarks supported students' reasoning in the placement of different fractions on the line, and provided an opportunity through productive confusion that fractions with different denominators can be placed on the same number line and the opportunity to build the concept that in decimals are

fractions partitioned into powers of ten. Some students were not showing that the benchmark fractions could be helpful in locating the tenths.

Implications for teaching – Using the benchmark fractions involving fourths allowed for students to recognize that we can put tenths and fourths on the same line, involving both decimals and fractions, yet also allowed for struggling with how to partition the whole into tenths when the line is already partitioned into fourths. Using the benchmarks was made more effective in this task because of the double number line, and vice-versa that the use of the double number line was enhanced by starting with benchmark fractions. Starting with benchmarks on a double number line allowed students to see that fractions and decimals are equivalent with numbers they are already familiar with ($\frac{3}{4} = 0.75$), and for some students in connection with the context of money.

Additional observations and implications for teaching

The double number line is clearly an excellent way for students to use their understanding of fractions on the number line to begin understanding what decimals mean and their connection to fractions. Referring to numbers to the right of the decimal as fractional parts throughout the year is something for teachers to consider. It is critical to draw out student thinking to determine their current levels of understandings in order to select appropriate tasks. Decimals are how you notate fractions within the place value system.